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A STUDY OF TECHNOLOGICAL CHANGE ON FLUID MILK FARMS IN THE EDMONTON MILK SHED, 1940-1964

by

WALTER HAESSEL

A THESIS
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The undersigned certify they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Study of Technological Change on Fluid Milk Farms in the Edmonton Milk Shed, 1940-1964," submitted by Walter Haessel, in partial fulfillment for the requirements for the degree of Master of Science.

ABSTRACT

The study of technology has evolved through many stages with numerous approaches being used. A frequent shortcoming in these studies has been a failure to precisely define technological change. Consequently, a change in the level of technology is frequently confused with productivity change. These two terms are differentiated in this study and a model is introduced that can be used to measure the rate of technological change. This model is independent of any assumptions regarding returns to scale and factor substitution.

Time series data, collected from a sample of dairy farms for the 25 year period, 1940-1964, were applied to this model in an effort to measure the rate of disembodied technical change and the rate at which cows have improved in quality during the 25 years. The rate of disembodied technical change was estimated at 1.05 per cent per year for the period 1940-1951, while cow quality was estimated to improve at the rate of 0.05 per cent per year for the same period. For the 1952-1964 period, cow quality improved at the rate of 0.2 to 0.45 per cent per year, while disembodied technical change was found to be unimportant.

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I. INTRODUCTION

PURPOSE

The primary purpose of this study is to investigate some of the factors influencing productivity on fluid milk farms in the Edmonton milk shed for the period 1940-1964. An attempt is made to quantify the extent of two types of technical change in this period. The first is technical change embodied in the form of cow improvement. The second is all other technical change which takes the form of a residual. It is the aim of this study to measure the contribution of these two types of technological change to productivity changes.

Economists have recently exhibited considerable interest in the quantification of changes in productivity and in influences contributing to changes in productivity. There have been a number of empirical studies, some of which are reviewed in Chapter II. Most of the investigations examined there have been based on macroeconomic data with very little disaggregation. A variety of techniques have been employed, but all studies lead to the conclusion that technological change is an important factor influencing changes in productivity over time.

PRODUCTIVITY

The word productivity connotes a multitude of things to economists. In this study, two types of productivity are discussed. These are individual factor productivities and the efficiency of a composite

bundle of inputs. Both types of productivity can be quantified by calculating output/input ratios, or the reciprocal of these ratios. Individual factor productivity ratios are simply the output divided by the individual input, while total productivity ratios are output divided by some systematically weighted aggregation of inputs.

Productivity indexes of both types are listed in Table 1. These series were derived from the data in Table 3 of the Appendix.[1] Columns 2 through 7 are single input productivity ratios. These were calculated by dividing the output of milk for the particular year by the input of the individual factors. The resulting series were then converted to base 100 in 1940. The total productivity index of column 8 was derived as follows. The aggregate input index consisted of a combination of the labor, total capital, and the number of cows. These were combined in the form

$$I = L^{.373}E^{.119}C^{.508}.$$

In this formulation, L represents labor, E represents capital, and C represents the number of cows. The exponents are the relative income shares of the various factors in 1964. These were derived from figures reported by McBain.[2] After aggregation, the input index was divided into the output of milk series and the result converted to base 100 in 1940 to obtain the total productivity index.

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[1] The units of the data presented in Table 3 of the Appendix are given in Chapter IV, along with the source of the data and method of calculation.

[2] B. J. McBain, Dairy Farm Business Summary on Production Costs and Earnings; May 1st, 1963 to April 30th, 1964, Alberta Dept. of Agriculture, (Edmonton: Farm Economics Branch, 1964), p. 4.

TABLE 1

PRODUCTIVITY INDEXES
BASE: 1940 = 100

Year (1)	Labor (2)	Equipment (3)	Total Capital (4)	Protein (5)	TDN (6)	Cows (7)	Total Productivity (8)
1940	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1941	121.7	66.1	104.8	112.1	107.2	101.9	106.8
1942	133.8	45.0	104.2	115.4	109.5	104.9	113.9
1943	145.1	28.0	89.3	97.0	102.4	100.8	112.7
1944	155.1	27.5	104.9	108.0	116.3	103.9	119.8
1945	151.1	22.1	91.2	102.9	125.5	100.4	114.5
1946	159.5	21.0	89.3	96.4	99.1	103.7	118.5
1947	164.5	20.8	78.8	96.4	114.0	110.0	121.7
1948	155.7	20.8	70.1	99.3	112.1	103.6	115.1
1949	177.4	19.4	63.2	95.5	108.0	106.2	118.1
1950	205.1	22.1	66.3	98.2	116.0	113.2	131.2
1951	188.8	17.9	68.0	115.5	124.1	114.3	127.8
1952	179.6	19.0	65.0	98.2	105.7	108.6	121.8
1953	182.6	22.0	67.7	97.8	101.9	110.9	124.5
1954	188.7	26.1	71.6	104.9	108.7	111.7	127.6
1955	175.6	25.0	64.0	97.6	98.2	103.3	117.9
1956	180.9	23.5	64.8	97.8	99.2	106.6	121.0
1957	197.8	22.5	63.1	101.6	101.5	104.6	123.9
1958	246.7	16.1	61.4	94.6	97.4	119.4	140.0
1959	248.5	11.8	56.6	100.2	103.5	122.0	148.0
1960	292.9	8.1	54.5	99.0	102.1	126.7	155.0
1961	316.6	7.0	53.4	89.9	99.2	128.9	160.4
1962	289.5	7.7	57.4	86.7	96.3	130.5	157.6
1963	327.7	9.0	62.8	97.5	104.0	136.6	156.3
1964	357.7	6.3	60.3	96.5	101.6	132.1	172.6

The problems with this type of analysis are numerous. The meaning of the individual factor productivity indexes (columns 2 through 7) is not at all clear. For example, if column 2 is taken as the labor productivity index,then all change in productivity is attributable to labor and none should be attributed to the remaining factors. Thus, all entries in columns 3 through 7 (Table 1) should be 100. Since they

are not, columns 2 through 7 are mutually inconsistent and probably all wrong. The problem is that every individual factor productivity index is based on an assumption of ceteris paribus with respect to the ratio of factors employed and the productivity of all other factors used. Since this assumption is not likely to be fulfilled, it is equally unlikely that such an index is a true measure of factor productivity. Taking a specific example, according to the labor productivity index (column 2), one hour of labor in 1964 was approximately 3-1/2 times as productive as one hour of labor in 1940. However, the author contends this increase is not attributable solely to any change in labor itself, but is largely a consequence of the large increase in equipment and total capital per manhour. (See Table 3 of the Appendix.) It may even be argued that a laborer taken from 1940 and transferred into 1964 conditions would be almost as productive (if not as productive) as a 1964 laborer, given a short adjustment period.[1]

This total productivity index is fraught with difficulties that range from problems of aggregation to problems of interpretation. In this example, the input indexes were aggregated geometrically. The specific method of aggregation used is equivalent to assuming a Cobb-Douglas production function. Thus, it is necessary to assume the dairy industry is operating in competitive equilibrium with factors receiving their marginal value product. In addition, constant returns to scale are implied since the assumed function is homogeneous of degree one. Furthermore, each factor is assumed to receive a constant

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[1]This is assuming labor is separate from management, which is the case in the labor statistic used in this study. No allowance is made for management.

share of income throughout the entire period examined.

While it can be argued that the assumption of constant returns to scale is not unrealistic in the dairy industry, it is extremely difficult to find any form of rationalization for the assumption of constant income shares for the 25 year period studied. The extent of factor substitution, particularly between equipment (or total capital) and labor (see Table 3, Appendix), suggests such an assumption is entirely unwarranted.

In addition to all the problems of aggregation, the correct interpretation of the result is not clear. A productivity index has been calculated that indicates the productivity of three factors (namely labor, total capital, and cows) has increased 72.6 per cent (Column 8, Table 1). However, it is obvious that this productivity increase would be larger if labor had been weighted relatively more heavily in the aggregation procedure. Conversely, the productivity increase would have been smaller had labor received a smaller weight. A still different result would be obtained if equipment was used instead of total capital as a factor of production in the input index. Thus, the results of such a procedure can be altered almost at will by simply deriving a different input index.

In summary, productivity is a many faceted concept which is not well defined. Changes in productivity can result from numerous sources. These are: returns to scale (increasing or decreasing), factor substitution, and technical change. However, only technical change is of primary interest in this study.

TECHNICAL CHANGE

An increase in productivity can be illustrated on an isoquant map for a two factor case (Figure 1). Assume that Q_1 and Q_1' are isoquants representing the same level of output with Q_1' occurring in a later period. An increase in productivity is indicated because the same level of output can be obtained with a smaller combination of factors, labor (L) and capital (K). This increase in productivity is the result of one type of technical change, namely disembodied technical change. Disembodied technical change is an increase in productivity resulting from improved

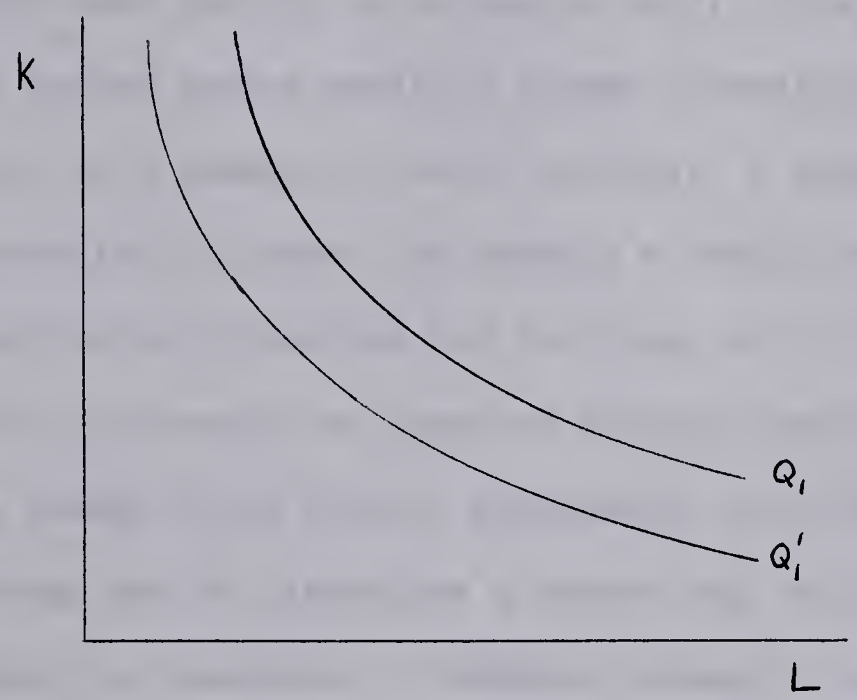

Fig. 1--Isoquant map showing technical change

productive techniques, superior knowledge, better management, etcetra. It is not the result of any improvement in the quality of productive factors because in Figure 1 homogeneity of factor input over time is implied since isoquants of successive periods are drawn on the same isoquant map.[1] The increase in productivity cannot be due to economies

[1]A discussion of this point can be found in G. L. Johnson, "A Note on Non-conventional Inputs and Conventional Production Functions," Agriculture in Economic Development, ed. C. Eicher and L. Witt (New York: McGraw-Hill Book Co., 1964), pp. 120-4.

of scale since total output is the same in both cases. Similarly, factor substitution is ruled out as a possible source of productivity increase because if a ray is drawn from the origin, Q_1' will always yield the same output as Q_1 from a smaller combination of inputs employed in exactly the same proportions.

A second type of technical change, and one not shown in Figure 1, derives from improvement of factor quality over time. If in two successive time periods, a fixed number of manhours are combined with a fixed amount of capital (the capital in period two being of the same value but not of the same quality as in period one), it is possible that output in the second period would be larger or smaller than in the first period because of a change in factor quality. A change in factor quality could be embodied in labor, for example a result of improved efficiency of labor through education and training; or it could be embodied in capital, for example as improved design of equipment; or embodied technical change could involve improvement of both factors.[1]

Technical change can be classified a second way, that is, it can be either neutral or nonneutral. Technical change is said to be neutral if it leaves the marginal rates of substitution between factors unchanged.

FACTOR SUBSTITUTION

Assume the situation illustrated in Figure 2 exists where (as previously) Q_1' equals Q_1 in amount of output, with Q_1' occurring in a

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[1]A more technical definition of embodied technical change is provided in H. A. J. Green, "Embodied Progress, Investment, and Growth," Amer. Econ. Rev., LVI (March, 1966), pp. 138-51.

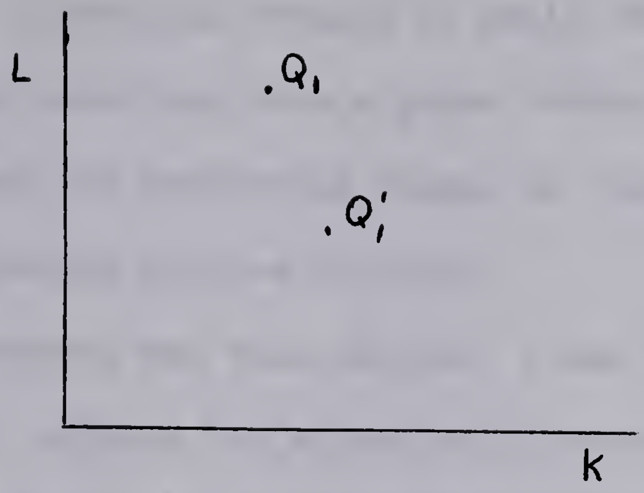

Fig. 2.--Factor substitution and productivity

later period. In this case, the productivity of labor has increased (on the basis of output/labor ratio) while, at the same time, that of capital has decreased.[1] The question arising from a situation such as this is, what has caused these productivity changes? One source of the increasing labor productivity could well be an increase resulting from an intensification of the capital to labor ratio. On the other hand, part of this increase in labor productivity could result from disembodied technical advance. The possibility of economies of scale is again excluded because total output is the same in both cases.

RETURNS TO SCALE

Returns to scale refers to the percentage change in output in relation to the percentage change in input when all factors are varied in the same proportion.[2] Returns to scale can be of three types: constant, increasing, and decreasing. Constant returns to scale are exhibited when percentage changes in output are equal to percentage

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[1]This is precisely what has occurred in the dairy industry, as can be seen from Table 1. The productivity of labor increased while that of capital decreased.

[2]This is the definition adopted for this study. The concept of constant proportionality is not a universally accepted criterion. For example, see E. H. Chamberlin, "Proportionality, Divisibility, and Economies of Scale," Quar. Jour. of Econ., LXII (Feb., 1948), pp. 229-62.

changes in inputs. Increasing returns to scale exist when the percentage change in output resulting from a given change in the quantity of inputs is larger than the percentage change in inputs, with the opposite being true for decreasing returns to scale.

Assume that between two time periods, t and t+1, technology remained constant.[1] Suppose the situation illustrated in Figure 3 exists, with output Q_t and Q_{t+1} being 100 and 200 units respectively. In addition, assume increases in inputs of labor and capital from

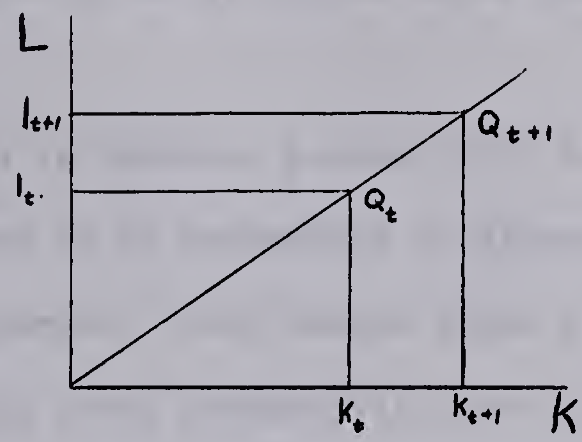

Fig. 3.--Increasing returns to scale

50 units in period t to 75 units in period t+1. Thus, economies of scale are present because the percentage increase in inputs has been only 50 per cent as opposed to a 100 per cent increase in the quantity of output.

THE PROBLEM OF SEPARATION

The problem of quantifying the extent of technical change really becomes one of separating a productivity change into its component parts, namely productivity changes arising from (1) factor substitution, (2) economies (or diseconomies) of scale, and (3) technical change which can be either embodied or disembodied. This can be illustrated by means

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[1] This assumption will be relaxed in the following section.

of a diagram (Figure 4). As before, suppose output increases from 100 to

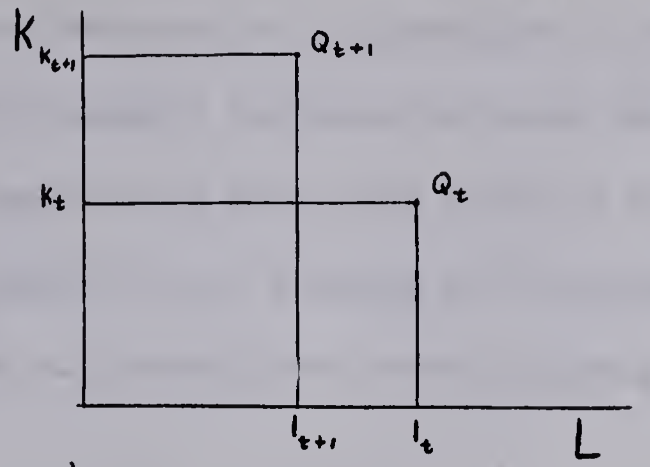

Fig. 4.--The components of productivity

200 units between periods t and t+1. Suppose further that the input of capital increases from 50 to 75 units, while the input of labor decreases from 70 to 60 units.

One conclusion is obvious; productivity has increased. From the information presented it is impossible to discuss why this increase of productivity has occurred. Both single input productivity ratios have increased, as has the total productivity ratio of any aggregated index of inputs. It is not clear whether economies of scale have contributed to the productivity increase. Nor is it clear whether factor substitution and/or disembodied technical change have contributed to the increase in productivity. It is even conceivable that diseconomies of scale are present but the effects of factor substitution and/or disembodied technical change are sufficient to outweigh the depressing effects of the diseconomies of scale.

The situation discussed above, though complex in itself, is simpler than that existing for dairy farms in the Edmonton milk shed for the 1940 to 1964 period. In this case, not all the single input productivities increased, as indicated by columns 2 through 7 on Table 1. The productivities of total capital and equipment decreased, while those for total digestable nutrients and protein remained relatively constant.

In a situation such as this, it is possible to obtain a total productivity index that actually shows a decrease in total productivity. This is accomplished by aggregating the input index in a manner that gives relatively more weight to the factors with declining productivity. Thus, it is not at all clear that there has been an increase in total productivity.[1]

A further complication arises when factors of changing quality are used as inputs in different periods. As indicated on page 6, homogeneity of factor input is implied by measuring inputs of successive periods on the same input axis. If this implicit assumption is relaxed (as indeed it must be in the real world), embodied technical change must be added to the list of potential contributors to increasing productivity. For example, it is well-known that dairy farmers have been able to improve the quality of their dairy herds through selective breeding, especially since the advent of artificial insemination. This fact would tend to indicate a contribution to productivity resulting from embodied technical change.

THE SCOPE OF THE STUDY

This study is limited in scope to the measurement of two sources of productivity change. These are (1) the rate of embodied technical change that has occurred in milk cows during the period 1940-1964, and (2) the rate of disembodied technical change during this period. It

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[1]A decrease in total productivity is obtained if an input index is formed by simply adding total capital and cows. While there is no rationale for such a combination, a result such as this is a possibility and a problem with index numbers that must not be overlooked.

can be argued that the measurement of embodied technical change should be extended to include all factors. This is true. The reason for limiting the study of embodied technical change to a single factor (cows) is the absence of a model that is suitable to measure embodied technical change in other factors. This problem will be discussed in greater detail in Chapters III and IV.

Chapter II provides a review of some of the models that have been employed to quantify technical change in other studies. These models are evaluated in terms of the present problem. Chapter III provides a description of the model utilized in this study. This model is basically a synthesis of several of the models reviewed. This is followed in Chapter IV by a discussion of the data, its strengths and weaknesses. Chapter V contains the results of the analyses along with a discussion of these results. In addition, some of the problems encountered in using this model are discussed in conjunction with the consequences of these problems. The final chapter provides a summary of the conclusions as well as a discussion of some general observations made during the study.

II. A REVIEW AND EVALUATION OF SEVERAL MODELS

The measurement of the contribution of technical advance to increases in aggregate output became popular about 1956. A major portion of the studies reported in the literature have been macroeconomic studies with entire sectors being involved. While the methods employed are not directly applicable to the problem considered here, a review of the methods is essential and provides the basis for the model outlined in the following chapter.

Two basically different approaches have been used in the study of technical change. These are the index number approach and the production function approach. Since this study follows the production function procedure, and since some of the problems involved with index numbers have been discussed in the introduction, only a brief discussion of this approach is provided here with the major portion of the chapter being devoted to the production function approach.

The study of technology has evolved through various stages, each stage being relatively more sophisticated than the preceding one. The historical development of sophistication is copied in this chapter. The simplest models are considered first and the more sophisticated last.

DISEMBODIED TECHNICAL CHANGE

The measurement of technical change began with the measurement of disembodied technical change. All the studies based on the index

number technique are in this category. Two index number techniques will be reviewed first and evaluated. This will be followed by a combined production function-index number technique.

The first technique to be reviewed, the constant dollar method, has been used in numerous studies.[1] The procedure involves weighting the inputs and outputs of each period (usually a year) in the time series with their prices for a given period. By adding the resulting constant dollar values of all inputs and outputs for each year, single input and output measures are obtained. The annual (total output)/(total input) ratios are frequently equated in interpretation to disembodied technical change.[2] However, such a measure is exactly analogous to the concept of productivity described in Chapter I. Many researchers, in fact, do not differentiate between the meanings of productivity and technical change and consequently use the terms interchangeably.[3]

The constant dollar method is based on a number of assumptions. First, the economic system in which the industry is operating is assumed to be in competitive equilibrium with factors being employed according to their value of marginal product. A linear production function is implied by the linear combination of the inputs. If various outputs are aggregated in the same manner to form one total output index, a linear

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[1]For example, see W. MacKenzie, "The Impact of Technological Change on the Efficiency of Production in Canadian Agriculture," Can. Jour. of Ag. Econ., X (No. 1, 1962), pp. 41-53.

[2]As defined in Chapter I.

[3]For example, this is true of MacKenzie, Ibid.

relationship between consumers' goods and attained satisfaction is assumed. In addition to these assumptions, all the problems of index number construction are inherent in the constant dollar method.[1]

The constant dollar method of analysis is closely related to the total productivity index presented in Table 1, with many of the problems outlined in Chapter I applying to this method. The problem of interpretation, combined with the multitude of problems arising from index number construction, resulted in the rejection of the constant dollar technique as a method of analysis for this study.

A technique very closely related to the total productivity index shown in Chapter I was employed by Urquhart in his study of the United States economy from 1850-1950.[2] Urquhart's model was based on three factors: land, labor, and capital. Letting T, L, and K represent land, labor, and capital respectively, he assumed a production function of the form $Q = L^a K^b T^c$, where $a + b + c = 1$. The exponents represented the share of income going to their respective factors, and were assumed constant for the entire period. These shares were derived by simply examining the allocation of income among the three factors. Urquhart then derived a joint input index (Table 2, line 4) by combining the

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[1]An excellent discussion of the problems of index number construction for this type of analysis is provided in S. H. Lok, An Enquiry into the Relationships Between Changes in Over-all Productivity and Real Net Return Per Farm, and Between Changes in Total Output and Real Gross Return, Canadian Agriculture, 1926-1957, Canada Department of Agriculture Technical Publication (Ottawa: Economics Division, 1961), pp. 22-44. Lok also provides a discussion of the assumptions underlying the constant dollar technique. See pp. 17-21.

[2]M. C. Urquhart, "Capital Accumulation, Technological Change and Economic Growth," Can. Jour. of Econ. and Pol. Sci., XXV (Nov., 1959), pp. 411-30.

three factors by means of the production function.[1] A general productivity index was obtained by dividing output[2] by the joint input series (Table 2, line 5). Urquhart called this "a measure of increased productivity by technological change (including economies of scale)."[3] It is not at all clear whether he considered economies of scale as part of technological change or whether he considered the increase in productivity was due in part to technical change and the remainder due to economies of scale.[4]

Urquhart's method of analysis to this point has been identical to that employed in Chapter I in the calculation of the total productivity index and is subject to the same problems outlined there. However, Urquhart extended his analysis beyond this point. He obtained indexes for individual factor productivities by dividing the output index by the individual factor inputs (Table 2, lines 7-9). These changes in factor productivities were the result of neutral disembodied technological change (by assumption)[5] and changing factor proportions.

From Table 2, Urquhart calculated a second table in which he

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[1]Land input is held constant over the entire period, labor input is based on manhours of work, and capital includes machinery, structures, equipment and inventories valued in real terms net of depreciation.

[2]Real net national product.

[3]Ibid., p. 420.

[4]His model implicitly assumed constant returns to scale. However, from the discussion accompanying the analysis, it is not clear whether he believed the restriction imposed by homogeneity of degree one to be realistic.

[5]Technical change is disembodied because no allowance is made for changing factor quality over time. The assumption of neutrality follows from the constant income shares.

attempted to identify the effects on factor productivities of technical change and changing factor proportions. The effect of technological change on the productivity of land (Table 3, line 1) was merely the general productivity index of Table 2 converted to an 1850 base. The effect that an increase in capital would have had on land productivity, had labor and land inputs and technology all remained constant is given in line 2 of Table 3. This was calculated by substituting values for land, labor and capital into the function $Q = L^a K^b T^c$, with the values for L and T being held at their 1850 level while the value for capital varied over time.

TABLE 2

INDEXES OF INPUT, OUTPUT, AND PRODUCTIVITY FOR THE WHOLE ECONOMY*
BASE 1929 = 100

	1850	1950
Input		
1. Labor	20	108
2. Capital	4.65	116
3. Land	100.	100
4. Joint	17.8	108
Output		
5. Output	5.47	182
Productivity		
6. General	30.7	168
7. Labor	27.4	169
8. Capital	117.6	157
9. Land	5.47	182

* Abbreviated version of Urquhart's Table I, Ibid., p. 419.

A corresponding procedure was followed for increases in labor (line 3). These were not changes in productivity that actually occurred, but rather productivity changes that would have occurred had factor proportions changed in the prescribed manner.[1] Line 4 gives the combined effect of capital accumulation, technical change and growth of the labor force on the productivity of land. This was calculated as the product of lines 1, 2, and 3.[2] A similar procedure was followed in the calculation of identical indexes for the effect of technical change and factor proportions on the productivity of labor and capital. These indexes are not reported here.

TABLE 3

INDEXES OF THE EFFECTS OF TECHNICAL CHANGE AND CHANGING FACTOR PROPORTIONS ON LAND PRODUCTIVITY*
BASE: 1850 = 100

Factor Influencing Productivity	1850	1950
1. Technological change	100	547
2. Increase of capital	100	178
3. Increase of labor	100	324
4. Combined effect	100	3327

*Abbreviated version of Urquhart's Table II, Ibid., p. 422.

The load of assumptions carried in Urquhart's procedure is heavy. Even if these assumptions can be tolerated, the meaning of the results

[1]These indexes merely exhibit the mathematical properties of the assumed Cobb-Douglas production function. If land, labor, and technology are held constant, capital accumulation has the same effect on the productivity of labor and land. The usefulness of such indexes is almost as questionable as their meaning.

[2]This index is simply line 9 of Table 2 converted to an 1850 base. It is not clear what this index means.

are in question. The same interpretation problems described in Chapter I for the total productivity index arise with Urquhart's method.

A slightly improved technique for separating disembodied technical change was introduced by Solow in his study of the private non-farm sector of the United States.[1] In this study, Solow defined technical change as "any kind of shift in the production function".[2]

Solow's model was based on a two factor production function of the general form

$$Q = F(K,L;t). \qquad (1)$$

Q, K, and L represent output, capital and labor respectively, and t for time allows for technical change. Beginning with the simpler case of neutral technical change, the production function takes the form

$$Q = A(t)f(K,L), \qquad (2)$$

where A(t) is an index of the cumulative effects of shifts over time. By differentiating with respect to time and dividing by Q this becomes

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}(t)}{A(t)} + A(t)\frac{\partial f}{\partial K}\left(\frac{\dot{K}}{Q}\right) + A(t)\frac{\partial f}{\partial L}\left(\frac{\dot{L}}{Q}\right), \qquad (3)$$

where dots represent time derivatives. By defining

$$W_k = \frac{\partial Q}{\partial K}\left(\frac{K}{Q}\right)$$

as the relative share to capital and $1 - W_k = W_l$ as the relative share to labor,[3] and substituting these values into (3) the result becomes

$$\left(\frac{\dot{Q}}{L}\right)\left(\frac{L}{Q}\right) = \frac{\dot{A}(t)}{A(t)} + W_k\left(\frac{\dot{K}}{L}\right)\left(\frac{L}{K}\right) . \qquad (4)$$

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[1] R. M. Solow, "Technical Change and the Aggregate Production Function," Rev. of Econ. and Stat., XXXIX (Aug., 1957), pp. 312-20.

[2] Ibid., p. 312.

[3] This follows from the underlying assumption of constant returns to scale.

In this form, $\frac{\dot{A}(t)}{A(t)}$ is an index of year to year technical change that can be estimated from time series data of Q, K, L, and W_k. Year to year changes can be used to approximate the time derivatives. In essence, this means that technical change in any one year is measured as the difference between two ratios. The first ratio is the observed relative change in the output/labor ratio, and the second is the relative change in the output/labor ratio that is caused by the relative change in the capital/labor ratio.[1]

The foregoing discussion has assumed neutral technical change. To test for neutral technical change (versus nonneutral) Solow derives the equation

$$(\dot{\frac{Q}{L}})(\frac{L}{Q}) = \frac{\dot{F}}{F} + W_k(\dot{\frac{K}{L}})(\frac{L}{K}). \qquad (5)$$

F is the same F appearing in (1). Solow argues it can be shown that if $\dot{F}/F$ is independent of K and L, then technical change is neutral.[2] After applying this test Solow concludes that technical change in the United States private non-farm sector has been neutral in the period from 1909-49.[3]

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[1] In a later article Solow indicates that if the share to capital (W_k) was a constant, the method would simply amount to determining how far the isoquants would have to be shifted neutrally so the observed (K,L) point would lie on it. R. M. Solow, "Reply," Rev. of Econ. and Stat., XL (Nov., 1958), pp. 411-13. If constant income shares were used, Solow's procedure would be identical to that of Urquhart outlined above. Thus, Solow's method can be considered an improvement of Urquhart's in that one less assumption is required; namely variable income shares to factors over time as opposed to the constant income shares assumed by Urquhart.

[2] Solow, "Technical Change and the Aggregate Production Function," p. 313.

[3] Ibid., p. 316. Resek has indicated that the test used by Solow can show neutrality in a nonneutral situation. The problem arises when

Solow used his index for technical change, A(t), to deflate the output series (i.e., he obtained an output series net of technical change) and fitted a production function net of technical change. This, in effect, was a production function holding technology constant. Other researchers have used similar approaches to investigate the effect of technological change on output for different sectors of the economy. For example, Massell[1] has applied this model to United States manufacturing from 1919-55. His findings were similar to those of Solow, as were the findings of Chandler who conducted a study using the same model to investigate productivity changes in the farm and non-farm sector of the United States.[2]

The three methods of measuring technical change discussed in this section are all relatively naive. Probably the best of the three is the one introduced by Solow. However, all three models require assumptions of (1) constant returns to scale, (2) homogeneity of factor inputs, and (3) competitive equilibrium in the economic system with factors being paid their marginal value products. The models are all designed to measure only disembodied technical change. If the assumption of factor homogeneity is violated, the measure of disembodied technical change

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technical change applies only to the K/L ratio in use and to no other ratio. Such a situation would be nonneutral but would appear neutral on the test. R.Resek, "Neutrality of Technical Progress," Rev. of Econ. and Stat., XLV (Feb., 1963), p. 57.

[1]B. F. Massell, "Capital Formation and Technological Change in United States Manufacturing," Rev. of Econ. and Stat., XLII (May, 1960), pp. 182-8.

[2]C. A. Chandler, "The Relative Contribution of Capital Intensity and Productivity to Changes in Output and Income in the U. S. Economy, Farm and Nonfarm Sectors, 1946-58," Jour. of Farm Econ., XLIV (May, 1962), pp. 335-48.

becomes biased upward by an influence that should actually be recorded as embodied technical change. If the assumption of constant returns to scale is violated, the result is a measure of productivity and not one of technical change.

Since the objectives of this study include the measurement of the rate of technical change embodied in cows as well as the rate of disembodied technical change, these models are clearly inadequate. The following section involves an evaluation of models that have been used for the quantification of the rate of embodied technical change.

EMBODIED TECHNICAL CHANGE

Salter introduced a model for measuring the extent of embodied technical change.[1] He differentiates technical knowledge into three levels: pure science, applied science, and applied production techniques. He points out that a different production function is associated with each level of technical knowledge. Thus, the dilemma arises as to which level of knowledge represents the technical advance that is of interest. Salter argues no businessman or engineer thinks in terms of which production function to adopt, but rather in terms of least-cost techniques. Once an investment in fixed capital is made the production function is no longer relevant and factor substitution becomes important. Thus, he concludes that the quality of new capital available is the deciding factor in decisions regarding the adoption of "best-practise" techniques.

Salter identifies four influences that determine successive period best-practise techniques. These influences (which follow) are elucidated

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[1]W. E. G. Salter, Productivity and Technical Change (Cambridge: Cambridge University Press, 1960).

with the aid of a diagram (Figure 5).

1. Technical advance is indicated on Figure 5 by the movement toward the origin of the successive isoquants,[1] t and t + 1, each of which represents one unit of output. The extent of technical advance from one period to the next is defined and measured by Salter as ". . . the relative change in total unit costs when the techniques in each period are those which would minimize unit costs when factor prices are constant".[2]

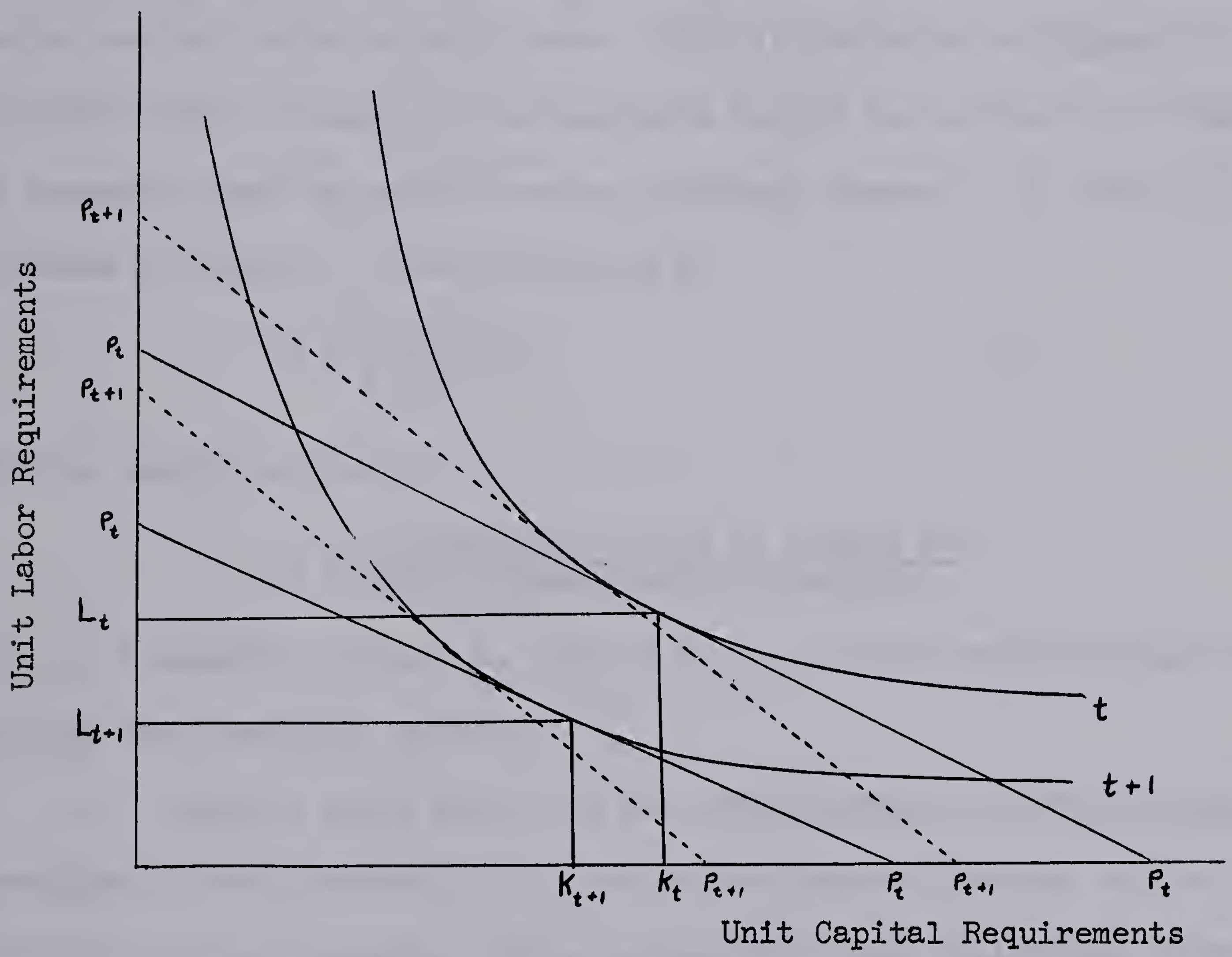

Fig. 5.--Successive best-practise techniques

In Figure 5, technical advance is defined as

$$T = \frac{L_{t+1}W_t + K_{t+1}G_t}{L_tW_t + K_tG_t} . \qquad (6)$$

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[1] Assuming constant returns to scale.

[2] Ibid., p. 30.

W_t and G_t represent the wage rate and the price of real investment in period t respectively, and L and K represent the amounts of labor and capital employed in the production of one unit of output. In other words,

$$T = \frac{\text{Cost per unit in period t+1 using period t prices}}{\text{Cost per unit in period t using period t prices}} .$$

(A different measure would result if this was framed in terms of period t+1 prices.)

2. The second influence described by Salter is disproportionate factor saving[1] which he calls bias. This is indicated on Figure 5 by the more rapid movement of the isoquants toward the ordinate (or what is commonly known as capital saving technical change). In terms of the notation on Figure 5, bias is defined as

$$B = \frac{K_{t+1}/L_{t+1}}{K_t/L_t} . \tag{7}$$

This is simply the ratio

$$B = \frac{\text{Capital/labor ratio of period t+1}}{\text{Capital/labor ratio of period t}} .$$

Bias is a measure designed to isolate changes in the capital/labor ratio arising from technical advance.

3. Salter's third factor in the determination of best-practise technique is the elasticity of substitution between factors, or the curvature of the isoquants. This is important when businessmen equate factor proportions to relative prices.

4. The final influence described by Salter is the changing relative prices of the factors.

Salter's model is based on several assumptions. Constant returns to scale is assumed. Technological advance must be embodied in new

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[1]This is defined as nonneutral technical change in Chapter I.

capital. Capital goods produced and adopted embody the latest of known technology. Capital goods in place do not share in the productivity increase arising from the increased efficiency that is embodied in new capital (i.e., no provision is made for disembodied technical change). The quality of labor is assumed to be homogeneous over time.

Salter defined technical advance at the margin in terms of the reduction in per unit real cost made possible through the adoption of new technology. This model, because of its meaningful definition of technological change, shows promise of being useful in future research. The reduction in per unit real cost is, as Salter has indicated, what the businessman or farmer hopes to achieve through the adoption of new technology. Unfortunately, the data used in the present study were not suitable for this type of model because the data were in physical units whereas Salter's method requires monetary units.

In 1959, Solow introduced a method of estimating capital embodied technical change[1] that is completely unrelated to the Salter method. Solow's measure is based on vintage production functions. For each vintage, v, of capital there is assumed to be a Cobb-Douglas constant returns to scale production function. These functions show the relationship between output at time t produced by capital of vintage v, $Q(v,t)$; the surviving capital of vintage v, $K(v,t)$; and labor working with capital of vintage v, $L(v,t)$. This function is of the form

$$Q(v,t) = Ae^{\lambda v}K(v,t)^{\alpha}L(v,t)^{1-\alpha}. \qquad (8)$$

λ represents the rate of capital embodied technical change and α and $1-\alpha$

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[1]R. M. Solow, "Investment and Technical Progress," <u>Mathematical Methods in the Social Sciences; 1959</u>, ed. J. K. Arrow, S. Karlin, and P. Suppes (Stanford: Stanford University Press, 1960), pp. 89-104.

represent elasticities of production of capital and labor respectively. Capital formed at time v is equal to gross investment, $I(v)$, and capital is assumed to depreciate exponentially at rate δ.[1] Thus, capital of vintage v at time t is defined as

$$K(v,t) = I(v)e^{-\delta(t-v)}. \qquad (9)$$

At any one time, the total capital stock will be the sum of capital of all vintages, which can be found by integrating over all vintages as

$$J_\lambda(t) = e^{-\delta t}\int_{-\infty}^{t} e^{v(\frac{\lambda}{\alpha}+\delta)} I(v)dv = \int_{-\infty}^{t} e^{(\frac{\lambda}{\alpha}v)} K(v,t)dv. \qquad (10)$$

Solow calls $J_\lambda(t)$ the effective capital stock. Total output in year t would then be given by

$$Q(t) = AJ_\lambda(t)^{\alpha} L(t)^{1-\alpha}. \qquad (11)$$

Labor represented by $L(t)$ is assumed to be homogeneous and distributed efficiently over all vintages of capital such that labor's marginal productivity is equalized on all equipment. Solow indicates that capital embodied technical change, λ, can be estimated from

$$\frac{\lambda t}{\alpha} + \text{constant} = \log\left(\frac{\Delta R + \delta R}{I}\right), \qquad (12)$$

where $$R = \left(\frac{Q}{L^{1-\alpha}}\right)^{\frac{1}{\alpha}}.$$

This method requires an exogenous estimate for the elasticity of production of capital (α).

Numerous assumptions are involved with this model. Technical change is assumed to be neutral. Constant returns to scale is assumed. Technical change must be embodied in new capital goods. The nature of technical change is such that at every point in time it affects only new

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[1]Solow indicates this implies the average life of capital is $1/\delta$ years. Ibid., p. 92.

capital goods (i.e., every capital good embodies the latest of known technology at the moment of its construction but it does not participate in subsequent technical progress). Labor is assumed to be homogeneous over time and efficiently distributed across all vintages of capital. Shares to capital and labor are assumed to be constant throughout the period. Finally, it is assumed capital embodied technical change is capital augmentive,(i.e., it has the same effect as an increase in the capital stock.).

For the purpose of making a conceptual comparison between Solow's disembodied and embodied technical change models, the disembodied model can be reformulated as

$$Q = Ae^{ut}K^{\alpha}L^{1-\alpha}. \qquad (13)$$

In this form, e^{ut} would be an exogenous shift function allowing for neutral disembodied technical change only. However, this imposes two additional assumptions over and above those required originally. Disembodied technical change is assumed to be neutral and income shares are assumed constant over the entire period. In summary, Solow's model for measuring the rate of embodied technical change is slightly more restrictive than his disembodied model since more assumptions are required. Only one form of technical change can be estimated, namely embodied technical change.

In 1962, Solow added a new feature to his embodied technical change model.[1] He explicitly introduced cyclical factors into the production function through the unemployment rate. This permitted him to differentiate

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[1] R. M. Solow, "Technical Progress, Capital Formation, and Economic Growth," <u>Amer. Econ. Rev.; Papers and Proceedings</u>, LII (May, 1962), p. 76-92.

between potential output and actual observed output. Potential output, $P(t)$, was defined as a function of the effective stock of capital and the available supply of labor, $L(t)$, or $P(t) = F[J_\lambda(t), L(t)]$. No explicit mention of technical change is required since it is already included in the effective stock of capital.

Observed output, $Q(t)$, is less than potential output because employment is less than the available supply of labor and because some capital stands idle. Actual output differs from potential output by a factor which is a function of the unemployment rate, E, or

$$Q(t) = f(E)F(J,L). \tag{14}$$

The function actually fitted was of the form

$$Q = AJ^{\alpha}L^{1-\alpha}10^{b+cE+dE^2}. \tag{15}$$

The unemployment function was $10^{b+cE+dE^2}$ and $AJ^{\alpha}L^{1-\alpha}$ was the production function of potential output.

Solow experimented with various improvement factors (λ) for capital and in this way derived alternative series for the effective capital stock. These various capital stock series were then used in fitting the production function. The criteria for the best estimate for λ were the goodness of fit and low standard errors of the coefficients.[1]

Each of the models discussed in this section have many underlying assumptions. All three models were designed to estimate the extent of capital embodied technical change. In addition to this, Salter provided

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[1]This model does not require an exogenous estimate for α but rather requires alternative exogenous estimates for λ. In the previous model, an exogenous estimate for α was required and λ was estimated implicitly.

a measure for nonneutral technical change. The possibility of productivity gains arising from factor substitution due to changing factor prices was also investigated by Salter. Unfortunately, Salter's model required data of a nature that was not available in the present study. The Solow models introduced a means of estimating the rate of capital embodied technical change. A modified version of his method, which is described in Chapter III, is used to measure the rate of cow improvement in this study.

EMBODIED AND DISEMBODIED TECHNICAL CHANGE

The foregoing models have considered the rate of either embodied or disembodied technical change exclusively, but never simultaneously. Objections to this approach have been widespread in the literature.[1] Perhaps the first effort to synthesize the two approaches was introduced by Phelps in the form of a growth model.[2] The model employed by Phelps is based on a Cobb-Douglas type of production function which is a blend of Solow's embodied and disembodied technical change models. Phelps' model was of the form

$$Q = Ae^{ut}J^{\alpha}L^{1-\alpha}. \qquad (16)$$

J represents Solow's effective capital stock (which implicitly includes embodied technical change); L represents labor which is distributed evenly over all vintages of capital; and e^{ut} allows for disembodied

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[1] For example, Solow, "Technical Progress, Capital Formation, and Economic Growth," pp. 76-7; M. Abramovitz, "Economic Growth in the United States; A Review Article," Amer. Econ. Rev., LII (Sept., 1962), p. 773; Solow, "Technical Change and the Aggregate Production Function," p. 312; and A. Smithies, "Discussion," Amer. Econ. Rev.; Papers and Proceedings, LII (May, 1962), p. 92.

[2] E. S. Phelps, "The New View of Investment: A Neoclassical Analysis," Quar. Jour. of Econ., LXXVI (Nov., 1962), pp. 548-67.

technical change where t is time and 100u is approximately the constant percentage rate of disembodied technical change per unit of time.[1]

Although Phelps combined the two Solow models to obtain one containing both embodied and disembodied technical change, he made no effort to estimate both rates of technical change simultaneously. The Phelps model was extended by Intriligator in two ways.[2] First, the rate of embodied and disembodied technical progress were estimated simultaneously. Second, the rate of technical progress embodied in improved quality of labor, as well as that of capital, was estimated.

The model used by Intriligator involved a constant returns to scale Cobb-Douglas production function estimating potential output, which was related to actual output by Solow's unemployment function as

$$Q = Ae^{b+cE+dE^2}e^{ut}J^{\alpha}M^{1-\alpha}. \qquad (17)$$

As before, J is the effective capital stock, e^{ut} is the time trend allowing for disembodied technical change, $e^{b+cD+dE^2}$ is the unemployment function, and M is the labor input index weighted for quality changes. The labor indexes employed by Intriligator were indexes reported elsewhere and not specific to the model. The estimating procedure was identical to that used by Solow in his 1962 paper and outlined above.

The assumptions necessary for this model are not as severe as those required by Solow's model. Intriligator's assumptions are as

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[1]This is accurate for small values of u. When u = 0.1, a constant percentage change of 10.52 per cent per unit of time is implied while 100u indicates 10 per cent. The accuracy of the relationship between 100u and constant percentage rate improves as u approaches zero.

[2]M. D. Intriligator, "Embodied Technical Change and Productivity in the United States 1929-1958," Rev. of Econ. and Stat., XLVII (Feb., 1965), pp. 65-70.

follows. Disembodied technical change is assumed to be neutral. Constant returns to scale is assumed. Labor is assumed to be efficiently distributed over all vintages of capital such that labor's marginal productivity is equalized on all capital. Finally, capital embodied technical change is assumed to be capital augmentive.

The Intriligator model was simply an extension of the Phelps model, which in turn was a synthesis of two earlier Solow models. The model in this study, and described in the following chapter, draws heavily from the Intriligator formulation of the embodied and disembodied technical change model. However, this model still contains two critical assumptions: returns to scale are constant and disembodied technical change is neutral. These problems are discussed in the remainder of this chapter. Several models are reviewed which make contributions to the method described in Chapter III.

TECHNOLOGICAL CHANGE AND RETURNS TO SCALE

The foregoing studies have all been based on the explicit simplifying assumption of constant returns to scale. Walters has indicated there is reason to doubt the appropriateness of this assumption.[1] The model used by Walters was a simple Cobb-Douglas with an exponential trend term of the form

$$Q = AK^{\alpha}L^{\beta}e^{ut}. \qquad (18)$$

Dealing with the non-farm private sector of the United States economy from 1909-1949,[2] Walters found the sum of $\alpha + \beta$ was significantly

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[1] A. A. Walters, "A Note on Economies of Scale," Rev. of Econ. and Stat., XLV (Nov., 1963), pp. 425-7.

[2] This is essentially the same data Solow used for his original article.

greater than one indicating increasing returns to scale. He suggested this would require a revision of Solow's results. Walters attributed some 27 per cent to 35 per cent of the increase in output to economies of scale in this period. Solow originally attributed this change in output to technological change, thus apparently biasing his estimates of technical change upwards.

In 1962, Brown and Popkin introduced a model designed to divide productivity changes in the United States into changes resulting from advances in neutral and nonneutral technology, and changes due to exploitation of economies of scale.[1] Their procedure involved dividing the entire period, 1890 to 1958, into sub-periods which they called technological epochs. A technological epoch was defined as a period in which no nonneutral technical change occurred. Within each of these technological epochs they divided productivity changes into weighted changes in input, economies of scale (if present), and neutral technical advance. This was done by the analysis of a production function.

The function employed was a Cobb-Douglas of the form

$$Q_r = A_r K_r^{a_r} L_r^{b_r} T_r^{c_r}. \qquad (19)$$

r indicates the particular epoch to which the function refers (i.e., a new function is fit to each technological epoch), and A_r, a_r, b_r, and c_r are all parameters referring to epoch r. T_r is a time index within epoch r.[2]

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[1]M. Brown and J. Popkin, "A Measure of Technological Change and Returns to Scale," Rev. of Econ. and Stat., XLIV (Nov., 1962), pp. 402-11.

[2]This is simply a version of the Solow disembodied model in which the assumption of constant returns to scale is relaxed. It is also analogous to the Walters model, the only difference being in the form of the trend term.

Each epoch was separated from other technical epochs as follows. A function was fitted to a period (say 1890-1905) in which there was assumed to be no nonneutral technical change. Then a tolerance interval was computed for observations beyond 1905 in which 95 per cent of the new observations fell with a probability of .95. If the forecasting error fell outside this range, the observations were no longer considered homogeneous with the 1890-1905 period and a new epoch was assumed.

Within each epoch, the influences of neutral technological change, economies of scale, and changing factor inputs on productivity were analyzed by taking the total differential of the production function, dividing by output, and grouping the resulting terms in a suitable fashion. The method is illustrated below.[1]

First, taking the total differential of (19) and dividing by output, the result becomes

$$\frac{dQ_r}{Q_r} = \frac{dA_r}{A_r} + \log K_r da_r + \log L_r db_r + \log T_r dc_r + \frac{a_r}{K_r}dK_r + \frac{b_r}{L_r}dL_r + \frac{c_r}{T_r}dT_r. \quad (20)$$

This model assumes a_r and b_r to vary over time in an unspecified manner. Once a_r and $b_r (r = 1,2, \ . \ . \ .,R)$ have been estimated it is possible to measure percentage change in output due to nonneutral technological change. Rewriting the second and third terms of (20) as[2]

$$T_{1r} = \log K_r \Delta a_{rs} + \log L_r \Delta b_{rs}; \quad (21)$$

where $s = r+1$, $\Delta a_{rs} = a_s - a_r$, and $\Delta b_{rs} = b_s - b_r$; then T_{1r} represents the percentage change of output due to nonneutral technological change.

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[1]The derivations are not shown. For an elaboration, see Ibid., p. 404.

[2]Using discrete changes as approximations to the differentials.

Assume increasing returns to scale exist and thus $a_r + b_r > 1$. By introducing the definitions $a^r + a_r/(a_r+b_r)$ and $b^r = b_r/(a_r+b_r)$, the excess of output accruing to firms operating in the region of increasing returns as opposed to those operating under constant returns can be shown as

$$E_r = (a_r - a^r)(K_r^* - \bar{K}_r)/\ \bar{K}_r + (b_r - b^r)(L_r^* - \bar{L}_r)/\ \bar{L}_r. \qquad (22)$$

K_r^* represents capital utilized in the last period (year) of epoch r; $\bar{K}_r$ represents capital used in the initial period of epoch r; the same applying to L_r^* and $\bar{L}_r$ for labor; and E_r represents the percentage change in output resulting from economies of scale in epoch r. The effect of decreasing returns can similarly be estimated.

Finally, it is possible to quantify neutral technological change in the r^{th} epoch by evaluating

$$T_{2r} = c_r(T_r^* - \bar{T}_r)/\ \bar{T}_r + \frac{\Delta A_{rs}}{A_r} + \log T_r \Delta c_{rs}. \qquad (23)$$

$\bar{T}_r$ and T_r^* indicate respectively the first and last periods of epoch r, s = r+1, and ΔA_{rs} and Δc_{rs} are defined as above.

Some of the foregoing studies have involved attempts to explain the change in output as a result of two things, a change in the quantity of inputs and a residual or trend. This residual has erroneously been termed technical change for lack of any better explanation of what has occurred.[1] Solow attempted to explain the productivity changes by arbitrarily weighting capital on the assumption that capital was becoming more efficient and thus more productive.[2] Denison attempted to explain

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[1]This is true of those studies which included only disembodied technical change.

[2]Solow, "Investment and Technical Progress" and "Technical Progress, Capital Formation and Economic Growth."

productivity changes by "dividing up" the residual among various factors such as increased productivity of labor through better working conditions and more education, productivity changes resulting from the advance of knowledge, et cetera.[1] While Denison's dividing up of the residual has been strongly criticized,[2] a similar approach has been used by Griliches in explaining productivity changes in the United States agricultural sector.[3]

The procedure employed by Griliches involved fitting a cross-sectional production function for 1940. The function fitted was an unspecified Cobb-Douglas[4] with six independent variables: livestock expense, other current expense, machinery, land, buildings, and man years of labor. Griliches found the sum of the exponents (1.362) was significantly larger than one, thus implying increasing returns to scale in 1940. By weighting the input series for quality changes that occurred in the 1940-1960 period (e.g., labor was weighted for educational changes and adjustments were made in some of the USDA indexes for biases resulting from underevaluation of quality changes) and allowing for the increasing returns to scale that existed in 1940, Griliches was able to explain all of the residual and a little more. (Griliches' weighting procedure was such that his estimated output for 1960 was larger than the actual

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[1]E. F. Denison, The Sources of Economic Growth in the United States and the Alternatives Before Us, CED Supplementary Paper No. 13 (New York: Committee for Economic Development, 1962).

[2]For example see Abramovitz, "Economic Growth in the United States: A Review Article," pp. 762-82.

[3]Z. Griliches, "The Sources of Measured Productivity Growth: United States Agriculture, 1940-60," Jour. of Pol. Economy, LXXI (Aug., 1963), pp. 331-46.

[4]That is, the sum of the exponents was not made to equal 1.

output in 1960, leaving him with a negative residual.) By keeping the same weights and adjusting the sum of the exponents to equal 1.2 (instead of 1.362 as estimated), the negative residual disappeared.

In summary, Griliches has employed a model that embodies technical change in the form of quality changes of all factors, not just capital and labor. In addition, the model allows for other than constant returns to scale, as do the Brown-Popkin and Walters models. The model introduced by Griliches implies all productivity changes must be embodied in the form of factor quality improvements, while just the opposite is true of the Brown-Popkin and Walters models in that all technological change is assumed to be disembodied. These models contribute to the model outlined in Chapter III since the model described there is based on an unspecified Cobb-Douglas production function.

NEUTRALITY OF TECHNICAL CHANGE

Of all the models examined in this chapter, only three have explicitly examined the problem of neutral versus nonneutral technical change. These are the first Solow model, the Salter model, and the Brown-Popkin model. All other models have been either based on assumptions of neutral technical change or the question has been ignored completely. Resek has objected to this approach.[1]

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[1]Resek, "Neutrality of Technical Progress," pp. 55-63. As mentioned above (p. 20), Resek has indicated the test for neutrality used by Solow could show neutral technical change in a nonneutral situation. The same objection applies to the study by Massell, "Capital Formation and Technological Change in United States Manufacturing." Consequently, Resek introduced a different test for neutrality. Essentially, this test involved an examination of the relationship over time between the capital/labor ratios and the marginal rate of substitution between capital and labor. The relationship of the scatter between these two variables would depend on the functional form of the production function and whether technological progress was neutral.

Resek also suggested if nonneutral technical change occurs, the relative causes of growth might have to be reevaluated and a third factor included; namely an interaction between technical change and capital. He indicated the possible importance of such an interaction by a hypothetical situation which follows.[1]

A_1 is the original isoquant for 100 units of output and X is the production point (Figure 6). Suppose a capital saving innovation takes place and the isoquant shifts to B_1. Since the inputs remain unchanged, production would continue at X but production would now be conducted on isoquant B_2 with output at 128 units. However, the K/L ratio in use is not the combination justified by the price ratio.

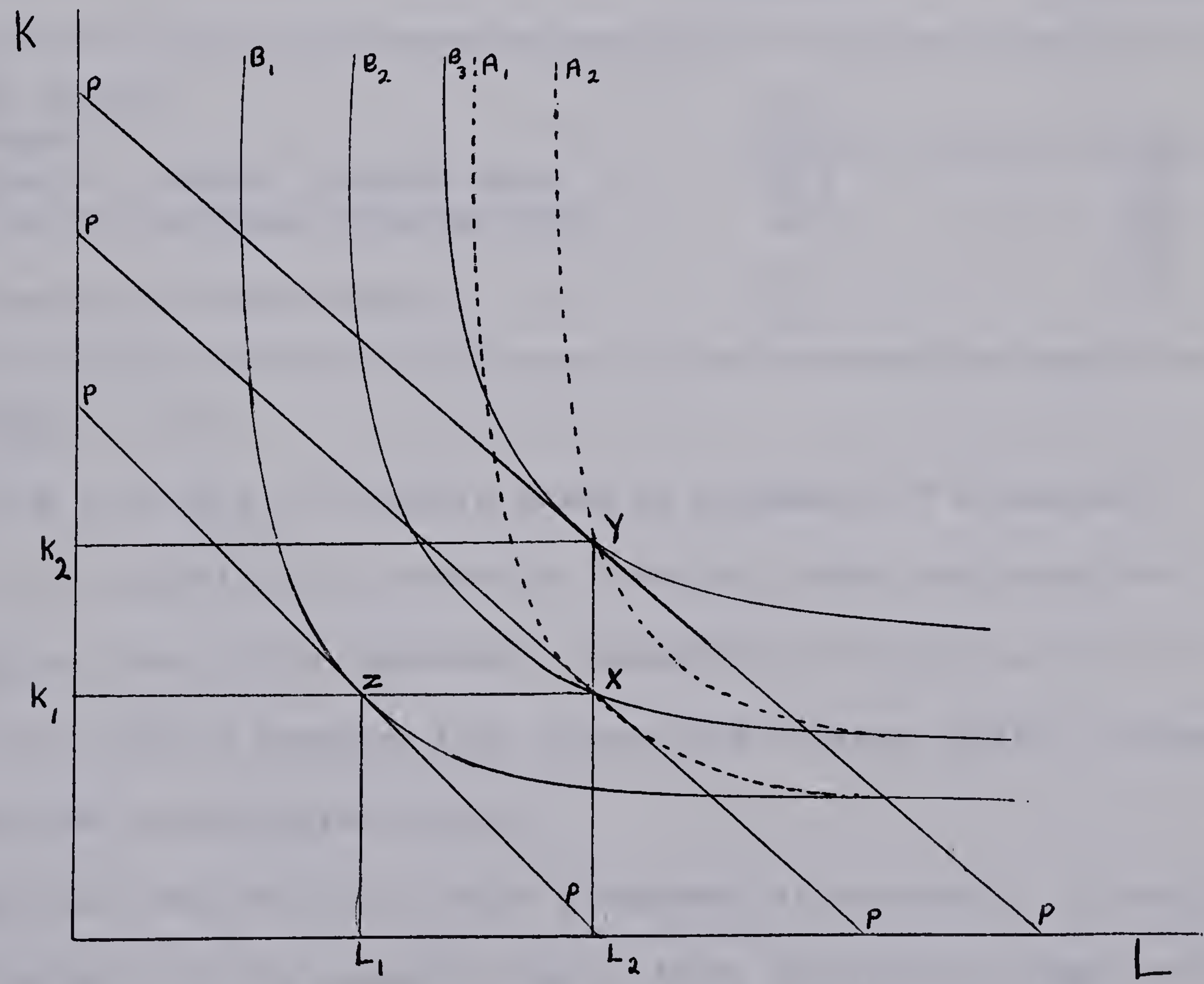

Fig. 6.--Technical change with increased capital

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[1]Resek, op. cit., pp. 56-7. Constant returns to scale are assumed throughout.

In order to achieve this equivalence of price ratio and slope of isoquant, capital could be increased to get to the point Y. This is on isoquant B_3 which has an output of 180 units. Production could also be carried on at point Y without technical change (i.e., on isoquant A_2 which implies 115 units).[1] The gains are tabulated in Table 4. The unexplained gain Resek attributes to the interaction between an increase in capital and technological advance.

TABLE 4

HYPOTHETICAL GROWTH IN OUTPUT*

	Output	Increase
Beginning output	100	
Final output	180	80
Changes due to capital increase only	115	15
Changes due to technical progress only	128	28
Sum	___	43
Unexplained gain (interaction)	___	37

**Ibid.*, p. 57.

Using this type of analysis based on isoquants of a constant elasticity of substitution production function, Resek concluded that nearly 30 per cent of the increased productivity of labor in the U. S. economy from 1919-59 resulted from interaction between capital increases and nonneutral technological change.

A closer examination of Resek's argument is necessary. It would appear the shift of the isoquant from A_1 to B_1 is actually labor saving and not capital saving as indicated by Resek. Suppose demand was such

[1]This point could be achieved by simply increasing the capital/labor ratio while the amount of labor employed remains constant.

that only 100 units of output could be sold. Suppose further the price ratio remained unchanged at pp. Thus, entrepreneurs would reduce the employment of labor by L_1L_2 until point z on isoquant B_1 was reached. Hicks has termed this labor saving technology.[1] By relaxing the assumption of limited aggregate demand, and assuming instead that demand is sufficient to consume all the output that can be produced with full employment of labor (L_2), different results are obtained. Profit maximizing entrepreneurs will employ L_2 units of labor and K_2 units of capital to operate at point Y on isoquant B_3. The increase in the capital/labor ratio is justified by a change in the marginal rate of substitution between labor and capital resulting from an increase in labor productivity through technical change.[2] Thus, the increase in capital is a result of technical change making it profitable to employ more capital. Resek attributes the increase in output resulting from this increase in capital to an interaction between capital increase and technical advance. However, it would appear this increase should be attributed to a capital increase only, the increase occurring at the new level of technology.[3]

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[1] J. R. Hicks, Value and Capital (2 ed.; London: Oxford University Press, 1965), p. 291.

[2] The increase in labor productivity must result from either disembodied technical change, labor embodied technical change, or both. This is because a change in the capital stock would be required if it was capital embodied. Thus, isoquant B_1 of Figure 6 would not be possible unless gross investment and gross depreciation left net investment at zero. This does not appear to be what Resek had in mind.

[3] This is because the change in technology occurred first. If the capital increase and productivity increase occurred simultaneously, the increase in labor productivity could result from capital embodied technical change which would cause the type of interaction Resek discusses. However, Resek specifically discounts embodied technical change. Resek, op. cit., p. 55.

In summary, Resek has argued that neutrality of technological change should not be assumed. In addition, he discussed the consequences of assuming neutral technical change when technical change was in fact nonneutral. While Resek's argument regarding the assumption of neutrality is valid, the discussion of the consequences of assuming neutrality when technical change was nonneutral is conducted at the expense of discounting embodied technical change. Yet, embodied technical change appears to be precisely the type of technical change with which Resek is concerned.

The important lesson to be learned from Resek's article appears to be the pitfall of assuming technical change is neutral. The possibility of a nonneutral shift occurring in the production function of a single industry (such as the dairy industry) is much greater than in a macroeconomic study such as Resek's. This is because one would expect nonneutral shifts in various industries would tend to offset one another in larger aggregations. Entrepreneurs would be expected to develop and adopt technological innovations that make more efficient use of underemployed resources, tending to neutralize shifts. This would tend to produce neutral technical change in aggregate data but the same argument does not apply to a single industry.

SUMMARY

An evaluation of several models used in the measurement of the rate of technical change has been presented in the current chapter. The model used in this study was developed from these models to a very large extent. The largest contribution to methodology must be attributed to the Solow models. These models included methods of measuring the rate

of embodied and disembodied technical change that were adapted and used in the present study. An alternative measure for the rate of disembodied technical change was suggested by Brown and Popkin. A means of estimating both the rates of embodied and disembodied technical change simultaneously was introduced by Intriligator. The ideas incorporated into the models of Brown and Popkin and Walters were useful in suggesting a means whereby the model used in this study was made independent of any assumption regarding returns to scale. Resek's suggestion of the importance of testing for neutrality of technical change led to the development of a test for a shift in the production function.

III. THE MODEL

The model employed in this study is a hybrid of several of the models reviewed. It is based on a Cobb-Douglas production function which makes allowances for both embodied and disembodied technical change. The model is independent of any assumption regarding returns to scale.

The basic function is of the form

$$Q = Ae^{ut}E^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}D^{\epsilon}. \qquad (24)$$

Q represents output of milk per farm per year, t represents a time index from 1 to 25 (for years 1940-1964), E represents the net stock of dairy equipment (excluding buildings),[1] L represents the labor per farm per year spent on the entire dairy operation, P represents protein fed to the dairy animals per year,[2] W(λ) represents the effective number of dairy cows on the farm, and D is a discontinuity variable.[3]

In defining the effective number of dairy cows it is assumed that (on the average) each cow remains in the herd for five lactations.[4]

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[1]Total dairy capital (K) was used as an alternative.

[2]Total digestible nutrients (N) was used as an alternative. Neither of these measures included feed value derived from pasture.

[3]A more extensive treatment of the data is provided in Chapter IV, including some necessary assumptions. For a summary of the coding, see Table 4 of the Appendix.

[4]The simple average of seven studies on the productive life of cows as reported by Winter is 4.94. This was rounded to 5.0 for convenience. See G. R. Winter, "Factors Influencing the Number of Dairy Cows in Iowa" (unpublished Master's thesis, Iowa State College, 1956), p. 30.

Thus, if the herd size remains constant, at any one time 1/5 of the cows would be in their fifth lactation, 1/5 in their fourth lactation, et cetera. Since the inherent ability of cows to produce milk over the period (1940-1964) has increased, a cow in her first lactation is "genetically superior" to a cow in her fifth lactation.[1] Thus, the younger cow should be weighted more heavily in a production function. By assuming the inherent ability of cows to produce milk has increased at a constant percentage rate, it is possible to weight these cows by the formula $e^{\lambda t}$, where λ is the rate of constant percentage improvement and t is a time index indicating the year of the study. In the t^{th} year, 1/5 of all the cows (C) would be in their first lactation and weighted as $\frac{C}{5}e^{\lambda t}$, 1/5 would be in their second lactation and weighted as $\frac{C}{5}e^{\lambda(t-1)}$, et cetera. The total effective herd would be defined by integrating over five years, or

$$W(\lambda) = \frac{C}{5}\int_{t-5}^{t} e^{\lambda t}dt. \qquad (25)$$

By using alternative values for λ, several series for the effective number of cows were derived. These series were then used in fitting time series production functions.

The trend term in the production function, e^{ut}, is a shift function that is designed to measure the rate of neutral disembodied technical change. This specification implies that disembodied technical advance proceeds neutrally at a constant percentage rate. The assumption of a constant percentage rate of disembodied technical change is an assumption that has served well in macroeconomic studies. An alternative

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[1]Assuming continuous improvement in cow quality over time.

specification of the form t^{θ} was tried. This form implies disembodied technical change proceeds at a diminishing rate if $0 < \theta < 1$, which is the range one would expect.

The discontinuity variable (D) is designed to test for stability of the production function over the period studied. Essentially, this variable consists of zeros for a number of years and ones for the remaining years. A coefficient (ϵ) is estimated for this variable the same as for the other variables. Significance of the net regression coefficient would then be interpreted as indicating a shift had occurred in the production function in the period of the change from zero to one in the discontinuity variable. This would suggest a nonneutral shift may have occurred at that time. Nonsignificance of this coefficient does not rule out the possibility of nonneutral technical change having occurred gradually over a period of years. It simply indicates that there has not been a sudden relatively large nonneutral shift in the production function. Nonneutral shifts that occurred over a period of years would not necessarily show up as a significant coefficient.

Several assumptions are necessary for this model. These are as follows. A cow is assumed to remain in the herd for five lactations on the average. The inherent ability of cows to produce milk improves at a constant percentage rate over time. Capital is assumed to be fully employed at all times. The quality of capital, labor, total digestible nutrients, and protein, (as measured in this study) are assumed to remain constant over the entire period (1940-1964). Some additional assumptions are dictated by the nature of the data. These will be outlined in Chapter IV along with a discussion of the data. The assumptions of homogeneity of capital and labor over the time period are

probably not realistic. However, no reasonable weighting procedure appeared to be available due to the nature of the data. An elaboration of the problems involved with alternative weighting procedures is combined with the discussion of the data in Chapter IV.

IV. THE DATA

SOURCE OF THE DATA

The data used in this study were obtained from the Farm Economics Branch of the Alberta Department of Agriculture with the kind permission of Mr. B. J. McBain. These data have been assembled continuously since 1939 for the Edmonton milk shed. The collection of these data was started in 1939 under the direction of Patterson of the Federal Government and carried on under his direction for three years. Since this time the studies have been supervised by McBain with the Provincial Government.[1]

As outlined by Patterson,

> The major objective of the study was to determine the factors affecting the earnings of dairy farm operators. More particularly it included the study of:
> (a) General organization and management of the farms.
> (b) dairy enterprise costs and returns.
> (c) Variations in costs and returns, and reasons for these variations.[2]

Total farm accounts and detailed dairy enterprise records were maintained along with physical data on feed and labor. In addition, a detailed livestock inventory was taken. These farm accounts, when

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[1] B. J. McBain, Economic Changes in Alberta Fluid Milk Production 1939-64, A paper presented to Agricultural Institute of Canada Convention, Vancouver June 21 to 25, 1965, (Edmonton, Farm Economics Branch, Alberta Dept. of Agric., 1965), p. 2.

[2] H. L. Patterson, Dairy Farm Business in Alberta: 1939 to 1943, Dominion Department of Agriculture Publication No. 812, Technical Bul. No. 67 (Ottawa: King's Printer and Controller of Stationery, 1948), p. 9.

combined with dairy records, were used to help farmers improve their dairy operation. In recent years these data have been collected primarily for the use of the Board of Public Utility Commissioners in setting the producer price of milk. However, the data have also been used to provide farm management service. Since these data were collected for a cost study rather than a study of productivity, they are not the ideal data for this study.

THE SAMPLE

In the first year of the study, records were completed on 47 whole milk farms in the Edmonton area. This was a 28 per cent sample of the Edmonton whole milk shippers.[1] In 1943, the study was extended to include other geographic regions and only 15 whole milk farm records were completed in the Edmonton area. This reduced sample[2] was continued until 1949 when a new sample was drawn to supplement the old sample.[3] The total sample was increased to 43. Since that time replacements have been sought to prevent the sample size from dropping below 28. An effort was made to obtain records from the same farms year after year if suitable records were available.[4]

To assure that a representative sample was drawn, the initial population was defined as those farms that had four or more cows

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[1] _Ibid._, p. 17.

[2] Between 15 and 20 farms inclusive.

[3] The accounting year used in this study runs from May 1 to April 30. To ease exposition, only the latest date will be listed in referring to any one accounting year. For example, the accounting year May 1, 1939, to April 30, 1940, will simply be referred to as 1940.

[4] For detail on the sample size see Table 1 of the Appendix.

milking and on which all or most of the income came from the dairy operation. (In actual fact, no farm ever had less than eight milk cows in any one year.) In later years, replacements were chosen in a manner such that the sampling densities for geographical regions were proportional to the concentration of fluid milk farms of various regions. In addition, an effort was made to keep the numbers of farms sampled per processor in proportion with the total number of farmers shipping to the various processors.

SOME ADDITIONAL ASSUMPTIONS

It was mentioned in Chapter III that some additional assumptions are necessitated by the nature of the data. These will be set out in the following sections.

FEED VARIABLES

The data contained minute detail on the physical quantity of various types of feed fed to the dairy herds. These numerous classes (e.g., oats, barley, alfalfa) were combined to form two variables. These variables were measured in hundred weights of digestible protein and total digestible nutrients. The feed compositions used in the aggregation were taken from Morrison[1] and are summarized in Table 2 of the Appendix.

Unfortunately, only a cash charge was available for the amount of pasture used by the dairy herds. Since the rates at which pasture was charged to the dairy have varied over time and these rates were not

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[1] F. B. Morrison, Feeds and Feeding; A Handbook for the Student and Stockman (22nd ed. rev. and unabridged; Clinton, Iowa: Morrison Publishing Co., 1959), Appendix, Table I.

available for the entire period, nutrients from this source were ignored. This necessitates an assumption, namely the proportion of protein and total digestable nutrients obtained from pasture remained unchanged over the period studied.

LABOR

Labor, measured in manhours, includes all the labor used on the dairy enterprise. This includes feeding and caring for replacement stock, et cetera. Hired and operator labor were weighted equally with a smaller weight being given to family labor, this weight depending on the age and sex of the family workers. Unfortunately, the labor variable did not include any measure of management time but only actual working time.

One of the assumptions stated in Chapter III is that labor is assumed to be homogeneous over time. It was also indicated that this assumption resulted in part from the nature of the data (or lack of data in this case). A procedure for weighting the labor series for quality changes similar to that used by Griliches was considered.[1] Griliches equated the level of education of farm labor to labor quality changes, and weighted the labor series accordingly. This assumes that a farm laborer with more education is more productive. While this may be true for agriculture as a whole, it is difficult to conceive a man with a high level of formal education being a more efficient cow milker than a laborer with a low level of education. Weighting labor for quality changes would appear to be a more appropriate procedure where the labor

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[1]Z. Griliches, "The Sources of Measured Productivity Growth: United States Agriculture, 1940-60,", p. 340.

variable included management, as was the case in Griliches' study. Furthermore, information regarding the level of education of dairy farmers in the Edmonton milk shed for the 25 years studied was not available.

In Chapter I it was argued that labor may not have changed in quality in this period. (This argument is based on a distinct separation between labor and management.) Given a reasonable period of adjustment, a laborer taken from 1940 and transferred onto a 1964 dairy farm would be as productive as a 1964 laborer, if no decision making by the laborer were involved. However, it is difficult to conceive a laborer on a modern dairy farm being removed from decision making entirely. Consequently, to be of equal productivity the laborers of 1940 and 1964 would have to possess equivalent decision making abilities. It is extremely difficult to separate (conceptually or practically) productivity changes arising from quality changes in labor itself and productivity changes arising from the availability of more equipment allowing labor to be used more efficiently and, thus, appearing to be of superior quality. Productivity changes of the latter type are not labor quality changes, but result from either capital embodied technical change, disembodied technical change, an increase in the capital/labor ratio, or some combination of these three sources.

CAPITAL

Two capital variables were used. These were equipment and total capital. Equipment included any tools, machinery, milking machines, etcetra that were used for the dairy operation. If a machine was used only part of the time for the dairy (say 50 per cent of the time) and

the remainder of the time for some other enterprise, only the dairy portion was recorded under dairy equipment. Total capital consisted of dairy buildings plus dairy equipment.

The dairy equipment series was derived by entering new equipment at its purchase price. The total equipment value was then depreciated at a constant rate of 15 per cent of the remaining balance. The total capital series was derived in much the same way, the only differences being in the rates of depreciation. The rates of depreciation used on buildings varied according to the type of structure and estimated length of life of the buildings.

No adjustments were made to the total capital or equipment series for either inflation of the original purchase price or quality changes. The data obtained from the Farm Economics Branch consisted of the net stock of equipment and total capital for each farm for the particular year. These figures were calculated by the staff of the Farm Economics Branch. Since the years of purchase of the capital goods were not recorded, it was not possible to deflate these purchase prices. Consequently, the capital series are in money terms rather than real terms. Conceptually, real capital is desired in a production function.

A method of estimating capital embodied technical change similar to that used by Solow was considered.[1] However, the nature of the sample precluded such a procedure. Solow assumed capital improved at a constant percentage rate. Beginning with the first year of the period studied, Solow derived artificial gross capital series by weighting new capital formed in each year and adding this weighted capital to the

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[1] Solow, "Technical Progress, Capital Formation, and Economic Growth."

existing capital stock. This artificial gross capital series was depreciated exponentially giving Solow his effective capital stock. This method was suitable for Solow's problem because he was concerned with the net capital stock of the economy. A decrease in the capital stock could result only if depreciation exceeded gross capital formation. In the present study the capital series (either equipment or total capital) were the average capital per farm as calculated from a sample. Thus, changes in the capital figures could result from two sources. First, a change in the stock of capital would result if net investment was other than zero. Second, a change in the capital value could be the result of sampling error.[1] While changes in capital stock values resulting from the latter source are acceptable in estimating a conventional production function, such a change is not acceptable to the estimation of a production function based on an effective capital stock series such as described by Solow. The problem arises not with the capital series itself, but in the relationship between the capital series and the other variables. If the sampling error is sufficiently serious, the values of the effective capital stock could be totally unrelated to the remaining variables.

In addition to the problem of sampling error, it is not clear if an exponential weighting procedure (such as that used by Solow) can be

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[1]A decrease in the total capital series was recorded for 1963 (Table 3, Appendix). In 1963, the sample contained only 29 farms compared to 33 the preceding year (Table 1, Appendix). Of the four farms in the sample in 1962 that were not in the sample in 1963, three farms recorded capital stocks that were substantially above the average recorded for 1962. If the 1962 total capital value is calculated using only the farms that were also in the sample in 1963, the resulting total capital value is less than the 1963 figure. (The figures used in this calculation are not reported here.)

applied to more than one variable without additional problems arising. A similar weighting method was used to estimate embodied technical change in cows.[1] An identification problem (similar to that encountered with multicollinearity) would probably arise if this method was used to quantify two rates of embodied technical change.

Since no other method of estimating quality changes embodied in capital appeared to be available, capital was assumed to be homogeneous over the period (1940-1964). However, it must be indicated that the capital values used, along with the assumption of homogeneity, are a grave deficiency in this study.

COWS

The observed number of cows was calculated by taking an average of the number of cows on hand per farm at May 1 and the following April 30. This provided a series of actual values to which the weighting procedure described in the previous chapter was applied to obtain values for effective cows.

MILK

The dependent variable used in the production function was the net milk produced per farm. This includes all milk marketed as fluid milk or cream, all milk used in the household, and all milk fed to livestock other than dairy calves.[2]

A problem of aggregation is present in the dependent variable.

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[1]This method is outlined in Chapter III.

[2]Milk fed to dairy calves was not included as this was part of the cost of production and, thus, is not part of the net milk produced.

Income derived from the sale of beef in the form of culled replacement heifers and steers or bulls theoretically should have been included as part of the dependent variable since the labor and feed used in the production of this beef is lumped with that required by the dairy cows and necessary replacements. On those farms on which it was considered to be a serious problem, budgeting was used to make adjustments.[1] With respect to the remaining farms, it is assumed the ratio of output of milk to herd credits has remained unchanged over the period studied.

CROSS-SECTION AND TIME SERIES

The data assembled are unusual in that most years contain sufficient observations to be useful as a cross-section.[2] In addition, these data are available for a sufficient number of years to be useful in time series analysis.

The model in the foregoing chapter was developed with the intention of using time series data. The time series were derived by averaging the cross-sectional observations for every year. This provided the basic time series data that were used for the model and are shown in Table 3 of the Appendix.

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[1]This was done at the time of data collection by the staff of the Farm Economics Branch.

[2]See Table 1 of the Appendix.

V. EMPIRICAL RESULTS

METHOD OF PRESENTATION

The results of many of the regressions are presented in the form of tables in the Appendix.[1] The regressions were fitted using a step-wise regression program that introduced one variable at a time. The variable introduced at each stage is that which accounts for the largest portion of the unexplained variation in the dependent variable.

On the heading of each table the model to which the coefficients pertain is indicated. All the models used are variations of the basic model outlined in Chapter III. The coding is summarized in Table 4 of the Appendix.

For each regression reported, the coefficients are significantly different from zero at the conventional 95 per cent level unless otherwise indicated.[2] Coefficients are reported only for those variables whose entry into the regression have significantly[3] reduced the unexplained sum of squares of the dependent variable remaining after the previous step in the regression.

Various levels were arbitrarily assigned to per cent improvement

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[1]The results of some regressions have not been reported. All tables discussed in this chapter are located in the Appendix.

[2]These non-significant coefficients are marked with an asterisk.

[3]Significant at the 95 per cent level.

in cow quality (λ). As a consequence of changes in λ, different variables significantly entered the regression. Thus, it is impossible to report these results in the form of a table without leaving gaps in the table. Sometimes entire columns are blank indicating that these variables were not significant for that particular specification of the model for any of the assumed values of cow improvement.[1]

For every regression presented the R^2 value is reported. In each case the value for the F test (on the ratio of the regression mean square to the error mean square at the final stage of the regression) leads to the conclusion that a similar reduction in the total sum of squares of the dependent variable would occur by chance with a probability of less than one in 1000. Thus, the F-values are not reported.

The lowest Student t-value[2] for each regression is reported. This is of interest in those regressions in which not all the coefficients differ significantly from zero and is also useful in the interpretation of the results.

The value for the assumed constant percentage rate of cow improvement is given in every regression in the form of λ. To convert λ to the equivalent constant percentage change, λ must be multiplied by 100 (i.e., $\lambda = .005$ implies a constant percentage change in cow quality of one-half of one per cent per year).[3]

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[1] For example, the blank column for β in Table 5 of the Appendix indicates that the labor variable did not significantly reduce the error sum of squares at any level of λ.

[2] The ratio of the regression coefficient to its standard error.

[3] As explained previously (p. 30) this approximation is fairly accurate for values of $\lambda < 0.1$.

The value of the constant coefficient (A) is presented in its naperian logarithmic form. All other coefficients are presented in the same units as they would appear in the model.

DISCUSSION OF RESULTS

The discussion in this section is concerned primarily with the discovery of the rate of disembodied technical change and the rate of cow improvement. Some comments are provided that may be of general interest, but these are limited in number.

THE 1940-1964 PERIOD

Six different combinations of the model described in Chapter III were applied to the period 1940-1964.[1] The results are presented in Tables 5 to 9 inclusive. Each of these tables will be examined in turn.

Underlying the model in this period is the implicit assumption that disembodied technical change is neutral. In addition, it must be pointed out that if both embodied and disembodied technical change are absent in the results of the regression, it must follow either that technology has been static over the period, or that the model has failed to quantify technical change. The latter appears to be the more realistic alternative.

MODEL 1: $Q = Ae^{ut}K^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}$ (Table 5)

When the assigned value of λ was zero, the coefficient indicating disembodied technical change (u) did not enter the regression at a significant level, thus, implying complete absence of technical change.

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[1]That is, six specifications were applied before the test for a shift was applied (discontinuity variable D).

Therefore, it appears this regression has failed to quantify technical change. If the value of λ is increased (at first to 0.0005 and then higher), the R^2 value increases and continues to increase until it reaches a peak when $\lambda = 0.0025$.[1] When λ is increased still further, the value for R^2 begins to decline. When λ assumes values from 0.0015 to 0.0025, the disembodied technical change coefficient (u) is negative (although not significantly different from zero). A negative coefficient is consistent with a decrease in the level of technology. The level of significance of the regression coefficients increased as the value of λ was increased from 0.0025 to 0.02, and began to decline for values of $\lambda > .02$.

On the basis of R^2, the best regressions are those with values of λ ranging from 0.0015 to 0.0035. Of these, in only one regression (that in which $\lambda = 0.0035$) were all the individual regression coefficients significantly different from zero. On the basis of the significance of the regression coefficients, the best regression was that with $\lambda = 0.02$.

MODEL 2: $Q = Ae^{ut}K^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$ (Table 6)

When this model involved no improvement in cow quality ($\lambda = 0.0$), disembodied technical change was estimated to proceed at the rate of 0.71 per cent per year (100u). As the value of λ increased, the R^2 value increased until it levelled off with $\lambda = 0.002$; remained constant while λ increased to 0.005; and finally declined for increasing values of λ 0.005. While all the regression coefficients were significantly

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[1]Part of the reason for this is that four independent variables are included in the regression as compared to three for the regressions in which λ is assigned other values.

different from zero, the level of significance of individual coefficients appeared to increase as the value of λ increased. On the basis of R^2, the best regressions were those with λ in the range of 0.002 to 0.005. Using as a criterion a high level of significance of regression coefficients, the higher the value λ assumed, the better the regression. Considering both criteria, the best regression was perhaps that which assumed λ somewhere in the vicinity of 0.005 to 0.01.

MODEL 3: $Q = AE^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}t^{\theta}$ (Table 7)

In this model, the highest R^2 and the second highest level of significance of the coefficients were obtained in the same regression, namely that regression which made no allowance for technical change ($\lambda = 0.0$ and t did not enter the regression), and therefore was of dubious value. More highly significant coefficients (but lower R^2) were obtained for values of $\lambda \geq 0.0025$, but once again technical change does not significantly enter the regressions, suggesting that the regressions were not meaningful. Thus, the best regressions appear to be those with λ ranging from 0.001 to 0.002.

MODEL 4: $Q = AE^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}t^{\theta}$ (Table 8)

This model exhibited the highest level of significance of regression coefficients in the regression without any technical change ($\lambda = 0.0$). The highest R^2 occurred in the regression in which $\lambda = 0.0005$. As the value of λ increased beyond 0.0005, both the value of R^2 and the level of significance of the coefficients decreased.

MODEL 5: $Q = AK^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}t^{\theta}$ (Table 9)

In this model there was also a regression without technical

change. Fortunately, this regression also exhibited the lowest R^2 value and the least significant coefficients. The highest R^2 and most highly significant coefficients also occurred in the same regression, namely that in which λ = 0.001. The best regressions appear to be those in which λ ranges from 0.001 to 0.002, if R^2 and a high level of significance of the coefficients are used as the criteria. Unfortunately, in this range the time exponent (θ) was negative.

MODEL 6: $Q = AK^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}t^{\theta}$

As indicated in the note under Table 9, the results of this model were identical to those of Model 2, with the exception of the regression in which λ assumed a value of zero. Since this regression falls into the class incorporating no technological change, no further discussion of this model is provided.

SOME OBSERVATIONS: 1940-1964

Labor generally did not enter these regressions at a significant level. In addition, whenever labor was in the regression capital was absent. This would suggest that labor is incompatible in the same regression with capital (measured either as total capital or dairy equipment). This is rather surprising, especially when one considers the simple correlation between E and L is only -0.6474 and between L and K is only -0.6605. Both these values are significantly different from zero at the 95 per cent level, but are considerably lower than the simple correlations between most of the remaining independent variables. Thus, multicollinearity should not be a problem.

The next observation also concerns the labor variable; namely, the negative labor coefficient that occurred every time labor entered the

regression. A glance at the data (Table 3) indicates that labor is the only variable with a downward trend. All the remaining variables exhibit an upward trend, and consequently, the simple correlations between labor and all the other variables are negative. However, this does not mean that the net regression coefficient would have to be negative. While on the basis of the overall trends a negative coefficient might be acceptable, economically it is surprising and indeed disturbing. A negative coefficient means that if labor had been increased in any one year, less milk would have been produced. While it does not follow that more labor applied to a farm would result in more milk (as would be implied by a positive coefficient),[1] such a conclusion would be more acceptable than the one that must follow from a negative coefficient.

Two possible explanations for a negative labor coefficient are offered. The first of these is that there was sufficient error in the labor variable to cause the sign to be the opposite of that expected. The second explanation is a more plausible one; namely, the assumption that labor was homogeneous over time has been violated. In other words, the productivity of labor may have changed as a result of a quality change in labor. The measure of labor (in manhours) has decreased while the actual contribution of labor to the production of milk may actually have increased or remained constant. At any rate, the contribution of labor could easily have been underestimated.[2] In actual fact, both factors may well have contributed to the negative coefficient.

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[1]All other inputs held constant.

[2]The problem of what constitutes a change in the quality of labor was introduced in Chapter I and discussed in detail in Chapter IV.

A final observation concerns the sensitivity of the model to alternative values of λ. The model appears to be very sensitive to changes in λ in the lower ranges, especially for $\lambda \leq 0.005$. For increasing values of λ over 0.005, there appears to be a steady trend (upward or downward) in some of the regression coefficients, but the same variables usually enter the regression. However, for values of $\lambda < 0.005$, different combinations of variables enter the regressions in the same model.

THE TEST FOR NEUTRALITY

In Chapter III, the test for neutrality that was described is a discontinuity variable which consisted of a sequence of zeros of varying length followed by ones for the remaining years. A significant regression coefficient for the discontinuity variable would indicate that a shift had occurred at the period in which the change was made from zero to one in the variable.

Discontinuity variables were added to Models 2 and 4. The value for λ was varied over the range from zero to 0.045, since this appeared to be the most sensitive range. In addition, the change from zeros to ones in the discontinuity variable was varied from 1943-44 to 1959-60. The regression coefficient became significant in the region of 1950-1954, with the sharpest break appearing to occur between the years 1951 and 1952.

This led to the conclusion that there was some type of shift in the production function in the period. Thus, the data was split into two groups, one group ranging from 1940 to 1951, inclusive, and the other ranging from 1952 to 1964, inclusive. Models similar to those just

examined were then applied to these periods.

THE 1940-1951 PERIOD

MODEL 7: $Q = Ae^{ut}K^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}$ (Table 10)

In this model, disembodied technical change (u) was estimated to proceed at the rate of 1.17 per cent per year when $\lambda = 0.0$. When λ was increased to 0.0005, the R^2 value increased as did the level of significance of the coefficients. The estimate of disembodied technical change (u) remained at 1.05 per cent per year. When λ was increased to 0.001, effective cows remained the only significant productive factor. The R^2 value dropped somewhat, but the coefficient was highly significant. When λ was increased to 0.002, dairy equipment entered the regression at a significant level and the R^2 value increased. The R^2 value continued to increase until it reached a peak when $\lambda = 0.0035$, and declined for values of $\lambda > 0.0035$. The level of significance of the coefficients increased continuously as λ increased until λ reached a value of 0.01. For values of $\lambda \geqslant 0.003$, the results were (surprisingly) identical with those for $\lambda = 0.0$.

MODEL 8: $Q = Ae^{ut}K^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$ (Table 11)

For this model, when $\lambda = 0.0$, the rate of disembodied technical change was estimated at 0.99 per cent per year. When λ increased to 0.0005, the R^2 value dropped slightly but the level of significance of the coefficients increased. This, in part, was due to fewer independent variables having entered the regression. When $\lambda = 0.0015$, effective cows was the only productive factor. When λ was increased to 0.0025, the R^2 value reached a peak, and as λ was increased still more, the value of

R^2 declined. The level of significance of the coefficients increased as λ was increased from 0.0025 to 0.004, and the level of significance decreased as λ assumed higher values. The best regressions were perhaps those with λ ranging from 0.0025 to 0.01.

MODEL 9: $A = Ae^{ut}E^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}$ (Table 12)

When λ assumed a value of 0.0, disembodied technical change was estimated to proceed at the rate of 1.57 per cent per year. When λ was increased to 0.0005, the rate of disembodied technical change decreased slightly to 1.05 per cent per year. However, the R^2 value and the significance of the coefficients both dropped relative to the regression in which $\lambda = 0.0$. With $\lambda = 0.001$, effective cows was the sole productive factor. When λ was increased to 0.0025, total digestible nutrients also entered the regression, although the coefficient was not significant. The R^2 value dropped and continued to drop until λ reached a value of 0.0065. For values of $\lambda \geq 0.0065$, total digestible nutrients was the sole productive factor with disembodied technical change being estimated at 2.20 per cent per year. Only three of the regressions included more than one factor of production. These regressions occurred when λ was set at levels between 0.0025 and 0.006. The best regression was perhaps that which assumed $\lambda = 0.0$.

MODEL 10: $Q = Ae^{ut}E^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$ (Table 13)

This model exhibited the same results as did Model 9 (Table 12) for values of $\lambda < 0.002$. When λ ranged from 0.002 to 0.005, both embodied and disembodied technical change were included. However, the total capital coefficient was negative at a significant level. For values of $\lambda \geq 0.0055$, the variable for effective cows no longer entered the regression at a

significant level. However, the negative coefficient remained associated with the capital variable, and consequently, the results of this model appeared to be of rather dubious value.

MODEL 11: $Q = AK^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}t^{\theta}$

The results of this model were identical to the results of Model 7 (Table 10) with the exception of $\lambda \leq 0.0005$. This model will not be discussed further.

MODEL 12: $Q = AK^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}t^{\theta}$

The results of the last model applied to this period were identical to the results of Model 8 (Table 11) with the exception of the regression in which $\lambda = 0.0$. The results of this regression are not reported or discussed.

SUMMARY: 1940-1952

When λ assumed values of 0.001 and 0.0015, the only productive factor appearing in the regressions at a significant level was the effective number of cows. This is true of all six models tried. The highest R^2 value was achieved with Model 10 when λ assumed values ranging from 0.002 to 0.005. Since Model 10 also had negative capital coefficients when λ was in this range (0.002 to 0.005), it would appear that the results in this period (1940-1951) were the best when $\lambda = 0.0$. Alternative estimates for rate of disembodied technical change in this period were 1.57, 1.17, and 0.99 per cent per year. In addition, it is interesting to note that models six through ten all exhibited identical results when $\lambda = 0.0005$. The rate of disembodied technical change in these regressions was estimated at 1.05 per cent per year.

THE 1952-1964 PERIOD

MODEL 13: $Q = Ae^{ut}K^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}$ (Table 14)

When $\lambda = 0.0$, the regression exhibited no technological change. As λ increased to 0.001, the R^2 value increased slightly, remained constant when $\lambda = 0.002$, and decreased slowly as λ continued to assume higher values. With the exception of $\lambda = 0.0$, the significance of the coefficients achieved the highest level when $\lambda = 0.001$. When $\lambda \geq 0.04$, the results of the regressions were identical to the results when $\lambda = 0.0$ and technical change was implied to be absent. The best regressions appeared to be those in which λ was assigned values ranging from 0.001 to 0.003.

MODEL 14: $Q = Ae^{ut}K^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$ (Table 15)

The results of this model were similar to those of Model 13 with the exception of when $\lambda = 0.0$. In that regression ($\lambda = 0.0$), disembodied technical change was estimated to proceed at the rate of 1.70 per cent per year. However, feed measured as protein was the only productive factor. The R^2 achieved its highest values when λ ranged from 0.002 to 0.0045. The regression coefficients were most highly significant when $\lambda = 0.0$ or when $\lambda > 0.07$. However, when λ assumed values in these ranges ($\lambda = 0.0$ or $\lambda > 0.07$), only two coefficients were estimated to be significant. Possibly the best regressions were those with λ ranging from 0.001 to 0.0045.

MODEL 15: $Q = Ae^{ut}E^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$ (Table 16)

When λ assumed a value of 0.0, the result was identical to the result of Model 14 (when $\lambda = 0.0$), with disembodied technical change being

estimated at the rate of 1.70 per cent per year and only one productive factor entering the regression at a significant level. When λ = 0.001, the R^2 value reached its highest level with a steady decline occurring as λ approached the value of 0.003, and with a slight increase being exhibited in the R^2 value as λ increased still further. When λ assumed values from 0.001 to 0.0025, three regression coefficients were estimated, the labor coefficient having a negative sign. When λ was outside this range (0.001 to 0.0025), only two coefficients were estimated and consequently these coefficients were significant at a much higher level.

MODEL 16: $Q = Ae^{ut}E^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$

This model exhibited the same result throughout the entire range of values assigned to λ. The results of these regressions did not make any allowance for technical change and hence are not reported.

MODEL 17: $Q = AK^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}t^{\theta}$

All the results of the regressions with this model were identical to the results of the regressions with Model 13 (Table 14).

MODEL 18: $Q = AK^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}t^{\theta}$

This was the last model applied to this period of time. The results obtained were identical to those of Model 14 (Table 15), with the exception of when λ was assigned a value of 0.0. This regression is not reported.

SUMMARY: 1952-1964

Of the six models applied to this section, only three exhibited different results. Two of these models (13 and 14) exhibited results

that were fairly similar. The best results of these two models were obtained when λ was assigned values ranging from 0.001 to 0.0045 in Model 14, and from 0.001 to 0.003 in Model 13. Regressions based on Models 14 and 15 estimated disembodied technical change at the constant percentage rate of 1.70 per cent per year. However, feed measured as protein was the only productive factor.

Model 15 was the only model in which labor entered the regression. Although the regression coefficients were not significantly different from zero, negative signs were associated with the labor coefficients, just as with the models applied to the 1940-1964 period. In addition, capital was absent when labor was present in the regression, as it was in the 1940-1964 regressions.

The results reported in Tables 14 and 15 appear to be more reliable than those of Table 16 since the results of Tables 14 and 15 were obtained by two separate models. In addition, Model 15 (Table 16) exhibits the disturbing negative labor coefficient.

AUTOCORRELATION OF THE RESIDUALS

TESTS FOR AUTOCORRELATION

One of the basic assumptions of least squares regressions is the serial independence of the residuals. This implies that successive residuals are drawn at random, independent of previous values.

Several tests for autocorrelation have been derived. One of the most common tests is the Durbin-Watson d statistic.[1] Durbin and Watson have tabulated upper and lower bounds that can be used to test for

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[1] J. Durbin and G. W. Watson, "Testing for Serial Correlation in Least Squares Regression II," Biometrika, XXXVIII (June, 1951), pp. 159-78.

significance of the d statistic. Exact significance levels for the d statistic have been tabulated by Theil and Nagar.[1] Unfortunately, both sets of tables only have significance levels for regressions involving 15 or more observations.

A closely related statistic is the von Neumann ratio. Ezekiel and Fox present a table that can be used to test the von Neumann ratio for significance.[2] This table contains values for four or more observations. However, no adjustment was made for including more than one independent variable in the regression. In the light of the Durbin-Watson and Theil-Nagar papers, this table does not appear to be applicable to problems involving more than one independent variable, in spite of the fact Ezekiel and Fox illustrate its use with problems involving two and three independent variables.

Ezekiel and Fox also present a table that can be used to test the serial correlation coefficient for significance.[3] Unfortunately, no adjustment is available for the number of independent variables in the regressions. Quenouille presents a table that can be used to adjust for sample size for regressions including up to four independent variables.[4] However, this test requires large samples involving at least ten observations for each independent variable.

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[1] H. Theil and A. L. Nagar, "Testing for Independence of Regression Disturbances," Amer. Stat. Assoc. Jour., LVI (Dec., 1961), pp. 793-806.

[2] M. Ezekiel and K. Fox, Methods of Correlation and Regression Analysis: Linear and Curvilinear (New York: John Wiley & Sons, Inc.), p. 341.

[3] Ibid., p. 338.

[4] M. H. Quenouille, Associated Measurements (New York: Academic Press Inc., 1952), p. 174.

A nonparametric test for serial correlation of residuals is available, based on the number of runs in the successive signs of the residuals. Tables that can be used to indicate the probability of the signs occurring at random have been prepared by Swed and Eisenhart.[1]

To test for autocorrelation of the regression disturbances for the twenty-five year period, the values tabulated by Theil and Nagar[2] for the appropriate numbers of independent variables at the 95 per cent significance level were used. Since no appropriate test for the 1940-1951 and 1952-1964 periods were available,[3] two tests were applied to the residuals of these periods. The Durbin-Watson and Theil-Nagar tables were independently extrapolated (graphically) to the appropriate numbers of observations and independent variables.[4] The results of these extrapolations are presented in Table 17 of the Appendix. These extrapolations are remarkably similar and do not deviate appreciably, no more so than do the tabulated Theil-Nagar and Durbin-Watson upper bounds. Since the results were so similar and the Theil-Nagar test is a more conclusive test, the extrapolated Theil-Nagar values were used to test for autocorrelation in the 12 and 13 year periods. The results of this test were then compared to the results of the nonparametric test based on the tables by Swed and Eisenhart.

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[1]F. S. Swed and C. Eisenhart, "Tables for Testing Randomness of Grouping in a Sequence of Alternatives," Annals of Mathematical Stat., XIV (March, 1943), pp. 66-87.

[2]Theil and Nagar, op. cit., p. 802.

[3]Due to insufficient observations in the regressions.

[4]The 95 per cent significance levels were extrapolated in both cases. The Durbin-Watson upper bound only was extrapolated.

THE RESULTS OF THE TESTS

THE 1940-1951 AND 1952-1964 PERIODS

The values calculated for the Durbin-Watson d statistic on the residuals of the regressions for the 1940-1951 and 1952-1964 periods were considerably above the extrapolated significance levels of the Theil-Nagar tables.[1] Assuming the extrapolated values are accurate, this would suggest the d statistic is not significant at the 95 per cent level and autocorrelation of the residuals is not a problem. This conclusion is substantiated by the nonparametric test for serial independence of the residuals. The Swed-Eisenhart tables indicated the signs associated with the residuals were ordered at random for every regression.

THE 1940-1964 PERIOD

The entire period analysis indicated the residuals were not serially independent for Model 3. The d statistic was highly significant at the 99 per cent level in Model 3 when λ was assigned values ranging from 0.001 to 0.002. For values of λ outside this range, the d statistic calculated on the residuals was not significant at the 95 per cent level.

The regressions for which the d statistics were significant were consequently recalculated after adjusting the data in the manner described by Smillie.[2] The results of these regressions are reported in

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[1]Table 17 of the Appendix. The calculated d statistic also exceeded the extrapolated Durbin-Watson significance levels in every case.

[2]K. W. Smillie, An Introduction to Regression and Correlation Department of Computing Science Publication No. 1 (Edmonton, University of Alberta, 1965), pp. 145-6. Essentially this adjustment is as follows. Suppose one is attempting to estimate the simple regression model

Table 18 of the Appendix.

THE CONSEQUENCES OF AUTOCORRELATION

While the tests have indicated that on the whole autocorrelation is not a problem, the tests conducted on the 1940-1951 and 1952-1964 periods cannot be considered reliable. The extrapolated values of the Theil-Nagar tables are certain to be subject to considerable error. The nonparametric test makes no assumption regarding the nature of the distribution of the residuals, and is not as sophisticated a test as might be used. Thus, it is wise to examine the consequences of significant autocorrelation for the model employed.

Johnston lists three main consequences of violating the assumption of serial independence of the residual.[1] These are as follows. First, estimates of the coefficients themselves will be unbiased but the sampling variances of these coefficients will be unnecessarily large. Second, the usual error formulas will likely underestimate the unduly large sampling variances of the least-squares coefficients. Thus, these error formulas will no longer be applicable. Finally, predictions based on a regression with autocorrelated residuals will have needlessly large sampling variances and thus be inefficient.

$Y = a + bX + u$, using n sets of observations (X_i, Y_i), $i = 1,2,\ldots,n$; with a and b representing parameters; and u_i representing the i^{th} residual. If serial correlation in the residuals is significant, the data are transformed in the manner

$$X_i^* = X_i - rX_{i-1} \text{ and } Y_i^* = Y_i - rY_{i-1};$$

where $$r = \left(\sum_{i=2}^{n} u_i u_{i-1}\right)/\left(\sum_{i=2}^{n} u_i^2\right).$$

The regression model is then refitted using the transformed data (X_i^*, Y_i^*), $i = 1,2,\ldots,n-1$.

[1]J. Johnston, Econometric Methods (New York: McGraw-Hill Book Co., Inc., 1963), p. 179.

The last of the three is of minimal importance in this study. However, the first two points are very important with respect to the model used, particularly the second one. This is because the "lowest t value" reported[1] and used in determining the "goodness" of alternative regressions is one of the statistics Johnston declares invalid if serial correlation is present in the residuals. This fact should be kept in mind when examining the conclusions of this study.

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[1]Tables 5 through 16.

VI. SUMMARY AND CONCLUSIONS

In this study an attempt was made to measure the rate of technological change on a sample of fluid milk farms. The model employed (Chapter III) was a synthesis of several macroeconomic models outlined in Chapter II. In addition, a test for shifts in the production function was added.

The model was designed to measure two types of technical change; first, that embodied in the form of cow improvement, and second, all other technical change or the residual. While the models discussed in Chapter II appeared to successfully fulfill their intent in the macroeconomic studies, the conclusions that can be drawn from the present study are not as precise and obvious as originally hoped. However, the following conclusions appear to be justified.

THE 1940-1964 PERIOD

1. Disembodied technical change was not important. This conclusion follows from the fact that Model 2 was the only one of the six that exhibited a positive coefficient for disembodied technical change. A lower percentage of the variance in the dependent variable was associated with variance in the independent variables for this regression ($\lambda = 0.0$ in Table 6) than for any of the regressions for Model 2. In addition, this regression exhibited one of the lowest percentages of variance in the dependent variable associated with variance in the

independent variables of any of the regressions of the 1940-1964 period. The unimportance of disembodied technical change in this period was not surprising considering the assumptions of neutrality and constant rate of disembodied technical change were violated, as was indicated by the shift in the function between 1951 and 1952.

2. The rate of constant percentage cow improvement was estimated to be about 0.3 to 0.35 per cent per year. The lower end of this range was fairly precisely indicated. Two models (1 and 5) resulted in negative regression coefficients when the rate of cow embodied technical change was assumed to be 0.25 and 0.2 per cent per year, respectively. Model 3 resulted in highly significant autocorrelation of the residuals when the rate of cow embodied technical change was assumed to be less than or equal to 0.2 per cent per year.[1] The upper bound on the range of the rate of cow embodied technical change was not as clearly defined. All the models indicated that a steadily decreasing portion of the variance of the dependent variable was associated with variation in the independent variables as the rate of cow embodied technical change was increased beyond a certain rate. This rate was 0.35 per cent per year in Models 1, 2, 5, and 6; and 0.05 per cent per year in Model 4. The ratio of the regression coefficients to their respective standard errors (Student t values) indicated that the level of significance of the regression coefficients increased as the rate of embodied technical change was assumed to increase beyond 0.05 per cent per year for Models 2 and 6. Models 1 and 5 indicated the highest level of significance of

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[1]These regressions were recalculated after adjustments for autocorrelation were made on the original data. The results of these regressions (Table 18, Appendix) suggest the rate of embodied technical change was larger than 0.2 per cent per year.

regression coefficients when cow quality was assumed to improve at the rate of 2.0 per cent per year. (This regression is not reported.)

THE 1940-1951 PERIOD

1. Disembodied technical change proceeded at the constant percentage rate of 0.99 to 1.57 per cent per year. If cows were assumed to be homogeneous during this period ($\lambda = 0.0$), alternative estimates of the rate of disembodied technical change were 1.17, 0.99, 1.57, and 1.57 per cent per year for Models 7, 8, 9, and 10, respectively.[1] The results of Models 7 and 8 were more convincing since these models included two independent factor variables as opposed to only one for Models 9 and 10.

2. Assuming all technical change must have been embodied in cow improvement, cow improvement was estimated to proceed at the constant percentage rate of 0.25 to 0.5 per cent per year. The portion of the variance in the dependent variable that was associated with variance in the independent variables was highest when the rate of cow quality change was assumed in the following ranges: 0.3 to 0.45 per cent per year for Models 7 and 11, 0.25 to 0.5 per cent per year for Models 8 and 12, and from 0.2 to 0.5 per cent per year for Model 10. However, Model 10 recorded a negative regression coefficient for total capital for all regressions in which the rate of cow improvement was assumed to be greater than or equal to 0.2 per cent per year. Therefore, the results of Model 10 should be discounted for rates of cow improvement exceeding 0.2 per cent per year.

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[1]Models 11 and 12 did not result in significant disembodied technical change regression coefficients. These models were of a specification that involved an assumption of a diminishing rate of disembodied technological improvement.

3. Assuming that technical change of both types was present in this period, embodied technical change was estimated to proceed at 0.05 per cent per year while the disembodied residual was estimated at 1.05 per cent per year. This result was the same for Models 6 through 10.

THE 1952-1964 PERIOD

1. Disembodied technical change in this period was not important. This was indicated by Models 13, 16, 17, and 18. Models 14 and 15 indicated that disembodied technical change proceeded at the rate of 1.70 per cent per year. However, in both these models only one productive factor (feed) entered the regression.

2. The rate of cow improvement was between 0.2 and 0.45 per cent per year. The results of Models 13 and 17 indicated the highest portion of the variance in the dependent variable was associated with variance in the independent variables in those regressions where cow improvement rates ranged from 0.05 to 0.45 per cent per year, while in Models 14 and 18 the highest portion was associated when the rate of cow improvement ranged from 0.2 to 0.6 per cent per year. In addition, the highest levels of significance of regression coefficients were recorded when cow quality improved at rates varying from 0.1 to 0.45 in Models 13 and 17, and at the rate of 0.2 per cent per year in Models 14 and 18.

SOME GENERAL COMMENTS

The conclusions are largely based on differences in the R^2 values between the various regressions. It must be pointed out that these differences were very small, in many cases differences being recorded only in the fourth significant digit. This fact should be kept in mind when examining the conclusions.

The models for the 1940-1964 period are based on two assumptions: first, disembodied technical change has been neutral and constant over time, and second, cow improvement has proceeded at a constant percentage rate over the entire period. Conclusions based on the results of the regressions for the 1940-1951 and 1952-1964 periods would tend to discredit these assumptions. Regressions for the 1940-1951 period provided estimates for disembodied technical change of .99 to 1.57 per cent per year with regressions for the 1952-1964 period exhibiting no significant disembodied technical change. This suggests the rate of disembodied technical change was not constant during this period.

The analysis of the 1940-1951 period resulted in perhaps the most acceptable results, namely regressions containing both embodied and disembodied technical change simultaneously.

The extremely high R^2 values recorded add a misleading sense of accuracy to the results. All R^2 values recorded were over 0.90. This would suggest that in every case over 90 per cent of the variance in the dependent variable was associated with changes in the independent variables. However, a good portion of this high R^2 is the consequence of a common and very pronounced trend in the dependent variable and all the independent variables except labor.

The sum of the regression coefficients varied inversely with the rate of assumed cow quality improvement (λ). As λ increased, the sum of the coefficients tended to decrease, most of the decrease occurring in the coefficient for effective cows (δ). This change in the size of the coefficient is to be expected due to the exponential weighting procedure employed to obtain the series for the effective number of

cows.[1] The extremely low sum of the coefficients should therefore not be interpreted as implying decreasing returns to scale of the order that would be implied by a conventional Cobb-Douglas function with the same low sum of exponents.

The values estimated for embodied technical change appear to be quite low. These values may well be underestimated. The quantity of milk produced by a cow is a function of, not only her inherent ability for milk production,[2] but also her environment. Feed plays an important role in this environment. No matter how much cow quality improves, if the feed input remains unchanged, very little, if any, increase in milk production will be recorded. However, even if cow quality remains constant, considerable gains in milk production can be realized if feeding methods and animal husbandry practises are improved. Thus, an increase in feed intake may be considered as a necessary, and sometimes sufficient condition for an increase in milk production. On the other hand, an improvement in cow quality may be a necessary but never a sufficient condition for any substantial increase in milk production. Thus, an improvement in cow quality may be explained in the regression by the

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[1] This is because the regression is fit in its logarithmic form. The exponent becomes the equivalent of a multiplicative coding factor in ordinary linear least squares regressions. For example, suppose the function to be fit is $Y = A(e^{2t}X)^{\alpha}$. This function would be fit in the form $\log_e Y = \log_e A + \alpha(2t + \log_e X)$. If t is a concommitant variable (such as time) that increases for successive observations of X, this has the effect of decreasing the size of regression coefficient (α) relative to the size it would be if the function fit was of the form $Y = A'X^{\alpha'}$. This influence on α is analogous to the influence on β_1 when fitting the simple linear regression $Y = \beta_0 + \beta_1(2X)$ instead of $Y = \beta'_0 + \beta'_1 X$.

[2] λ is designed to measure changes in this ability as embodied technical change.

feed increase that must accompany a production increase.

SUGGESTIONS FOR FURTHER RESEARCH

In this study, an attempt has been made to measure the rate of disembodied and cow embodied technical change. Ideally, the model should be extended to involve a measure of the quality changes in all factors and not just cows. In this way, the critical assumptions of homogeneous factors over time could be relaxed.

In the foregoing section it was indicated that the R^2 values were at an extremely high level; probably too high for meaningful interpretation. A fitting of these same regressions by using first differences, instead of the actual data, would tend to lower these R^2 values to a range where they would be more meaningful. It would be interesting to compare the conclusions drawn from first difference regressions with the conclusions presented in this chapter.

It was indicated in Chapter IV the data was not ideal for this type of study. The problems encountered with the labor variable are troublesome. It is extremely disturbing to see labor playing such a minor role in this study. Some allowance for management in the measure of labor would probably go a long way toward resolving the problems of negative or nonsignificant regression coefficients for labor. More specific data on pasture would also be useful since another assumption could possibly be relaxed.

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APPENDIX

TABLE 1

NUMBER OF FARMS SAMPLED

Year	No. of Farms	Year	No. of Farms	Year	No. of Farms
1940	47	1948	16	1957	33
1941	45	1949	43	1958	32
1942	42	1950	41	1959	33
1943	15	1951	37	1960	30
1944	20	1952	34	1961	34
1945	19	1953	33	1962	33
1946	17	1954	30	1963	29
1947	16	1955	33	1964	28
		1956	32		

TABLE 2

FEED ANALYSIS USED IN AGGREGATION OF VARIABLES[a]

Feed	Per cent Protein	Per cent TDN	Feed	Per cent Protein	Per cent TDN
Oats	9.4	70.1	Other purchased conc.	20.0	75.0
Barley	10.8	73.2	Other grains	10.0	76.5
Wheat	13.3	80.7	Alfalfa hay	10.9	50.7
Mixed grain	9.7	72.0	Oat greenfeed	4.9	47.3
Chop	9.1	72.2	Alfalfa-grass hay	7.5	50.0
16% dairy ration	16.0	72.4	Tame grass hay	5.3	49.3
32% dairy ration	32.0	69.0	Clover hay	8.1	53.2
Calf meal and pellets	20.0	72.0	Straw (oat)	0.7	44.8
Milk replacer	16.5	81.5	Wild hay	0.9	43.5
Oilcake and meal	30.5	75.6	Slough hay	4.1	48.0
Shorts and bran	13.5	66.0	Other hay	5.0	49.0
Screenings	9.2	67.2	Oat silage	5.0	47.0
Dried brewers grains	19.0	65.0			

[a]Source: Frank B. Morrison, Feeds and Feeding; A Handbook for the Student and Stockman (22nd ed., rev. and unabridged; Clinton [Iowa]: Morrison Publishing Co., 1959), Appendix, Table I.

TABLE 3

TIME SERIES DATA 1940-1964

Year	Labor	Equip-ment	Total Capital	Protein	TDN	Cows	Milk
1940	5392.	81.	2054.	2012.	14197.	20.851	163745.
1941	4265.	118.	1887.	1728.	12748.	19.689	157587.
1942	4253.	190.	2080.	1840.	13691.	20.976	172817.
1943	4227.	329.	2618.	2361.	15784.	23.533	186346.
1944	4247.	359.	2391.	2276.	14914.	24.500	199975.
1945	4412.	454.	2785.	2418.	15037.	25.684	202503.
1946	4501.	515.	3063.	2779.	19067.	26.765	218005.
1947	4433.	527.	3527.	2823.	16861.	25.625	221501.
1948	4590.	516.	3886.	2687.	16803.	26.688	217105.
1949	3885.	534.	4155.	2694.	16803.	25.698	209268.
1950	3628.	507.	4275.	2828.	16894.	25.415	226017.
1951	3885.	615.	4107.	2369.	15562.	24.811	222736.
1952	3965.	563.	4174.	2705.	17734.	25.353	216213.
1953	4068.	507.	4177.	2836.	19207.	25.909	225618.
1954	4117.	449.	4156.	2779.	18933.	27.033	237163.
1955	4289.	453.	4485.	2881.	20202.	28.182	228717.
1956	4441.	514.	4723.	3065.	21305.	29.156	244046.
1957	3897.	515.	4658.	2831.	20015.	28.485	234141.
1958	3648.	840.	5587.	3551.	24330.	29.125	273256.
1959	3830.	1208.	6405.	3545.	24227.	30.152	289016.
1960	3630.	1960.	7432.	4009.	27424.	32.433	322885.
1961	3599.	2443.	8129.	4731.	30240.	34.176	346085.
1962	4215.	2374.	8105.	5253.	33378.	36.152	370646.
1963	4003.	2195.	7961.	5021.	33219.	37.138	366323.
1964	3668.	2114.	8292.	5073.	34011.	38.393	398402.

TABLE 4

CODING FOR THE MODEL

Symbol	Derivation	Meaning	For elaboration see pages
Q	Observed	Pounds of milk per farm	42, 53-4
E	Observed	Dairy equipment investment	42, 50-53
N	Observed	Cwt. of total digestible nutrients per farm	42, 48-9
P	Observed	Cwt. of protein per farm	42, 48-9
C	Observed	Number of cows	42-3, 53
D	Imposed	Discontinuity variable	42, 44
$W(\lambda)$	Calculated	Effective number of dairy cows	42-3
λ	Imposed	Assumed constant percentage cow improvement	42-3
u	Estimated	Constant percentage disembodied technical change	42-4
t	Observed	Time index	42-4
α	Estimated	Elasticity of production of equipment or total capital	42
β	Estimated	Elasticity of production of labor	42
γ	Estimated	Elasticity of production of protein or TDN	42
δ	Estimated	Elasticity of production of effective cows	42
ϵ	Estimated	Discontinuity coefficient	42, 44
A	Estimated	Constant coefficient	42
θ	Estimated	Time exponent	42-4
K	Observed	Total dairy capital stock	42, 50-53

TABLE 5

REGRESSION COEFFICIENTS FOR THE MODEL $Q=Ae^{ut}K^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}$
1940-1964

λ	$\log_e A$	α	β	γ	δ	u	R^2	lowest "t" value
0.0	6.656	.0944		.4168	.3032*		.9842	1.51
0.0005	8.032	.0899		.2378*	.4191		.9872	2.03
0.0015	10.07	.1138			.4640	-.0062*	.9885	-1.94
0.002	10.29	.1228			.3787	-.0067*	.9884	-2.05
0.0025	9.217	.1201		.1596*	.2353	-.0053*	.9890	1.34
0.0035	8.844	.1156		.2390	.1101		.9875	2.11
0.006	8.755	.1188		.2620	.0627		.9871	2.36
0.01	8.687	.1206		.2765	.0367		.9869	2.53
0.03	8.601	.1221		.2928	.0177		.9866	2.54
0.09	8.534	.1222		.3023	.0037		.9864	2.46

*The coefficient is not significantly different from zero at the conventional 95 per cent level but the inclusion of the last variable in the regression has significantly reduced the unexplained variance (at the 95 per cent level) of the dependent variable remaining after the previous step of the regression.

TABLE 6

REGRESSION COEFFICIENTS FOR THE MODEL $Q=Ae^{ut}K^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$
1940-1964

λ	$\log_e A$	α	β	γ	δ	u	R^2	lowest "t" value
0.00	7.973	.0636		.4896		.0071	.9822	2.44
0.0005	8.448	.0656		.2279	.4981		.9874	2.12
0.001	8.867	.0739		.2251	.3561		.9895	2.41
0.0015	9.008	.0784		.2406	.2649		.9901	2.75
0.002	9.068	.0810		.2539	.2085		.9902	2.99
0.0025	9.097	.0827		.2642	.1710		.9902	3.17
0.0035	9.124	.0846		.2780	.0846		.9902	3.41
0.005	9.136	.0861		.2900	.0890		.9902	3.61
0.006	9.134	.0867		.2951	.0745		.9901	3.69
0.01	9.143	.0879		.3059	.0449		.9900	3.87
0.03	9.138	.0890		.3179	.0148		.9899	4.06
0.06	9.128	.0890		.3223	.0072		.9898	4.12

TABLE 7

REGRESSION COEFFICIENTS FOR THE MODEL $Q=AE^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}t^{\theta}$
1940-1964

λ	$\log_e A$	α	β	γ	δ	θ	R^2	lowest "t" value
0.0	8.519		-.2767	.3731	.7489		.9803	-2.88
0.0005	9.622		-.2137	.2285*	.6702		.9788	1.51
0.001	9.340	.1314*			.5551		.9727	1.96
0.0015	7.194	.1539		.3199*	.2066*		.9729	1.56
0.002	6.801	.1671		.3753	.1294*		.9718	1.22
0.025+	5.234	.2031		.5516			.9698	3.34

*See Table 5 for explanation.

+For values of $\lambda \geqslant .0025$ the results are identical.

TABLE 8

REGRESSION COEFFICIENTS FOR THE MODEL $Q=AE^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}t^{\theta}$
1940-1964

λ	$\log_e A$	α	β	γ	δ	θ	R^2	lowest "t" value
0.0	8.929		-.2545	.4577	.5788		.9815	-2.71
0.0005	9.347		-.2002	.3792	.4916		.9859	-2.41
0.001	9.319		-.1818	.4303	.3255		.9857	-2.15
0.002	9.211		-.1741*	.4961	.1782		.9846	-1.98
0.004	9.114		-.1727*	.5432	.0899		.9837	-1.90
0.009	9.046		-.1731*	.5731	.0393		.9830	-1.86

*See Table 5 for explanation.

TABLE 9

REGRESSION COEFFICIENTS FOR THE MODEL $Q=At^{\theta}K^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}$ [a]
1940-1964

λ	$\log_e A$	α	β	γ	δ	θ	R^2	lowest "t" value
0.0	6.656	.0944		.4168	.3032*		.9842	1.51
0.0005	8.032	.0899		.2378*	.4191		.9872	2.03
0.0010	9.919	.1211			.5051	-.0326	.9891	-2.17
0.0015	10.25	.1314			.3817	-.0319	.9888	-2.11
0.002	10.45	.1385			.3049	-.0313*	.9883	-2.02
0.0025	8.889	.1123		.2216*	.1547		.9877	1.93
0.003+								

[a]The results for a model of the form $Q=At^{\theta}K^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$ are identical to those presented in Table 6 with the exception of $\lambda = 0.0$.

*See Table 5 for explanation.

+For values of $\lambda \geqslant .003$ the results are identical to those of the model presented in Table 5.

TABLE 10

REGRESSION COEFFICIENTS FOR THE MODEL $Q=Ae^{ut}K^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}$
1940-1951

λ	$\log_e A$	α	β	γ	δ	u	R^2	lowest "t" value
0.0	8.658	.0854		.3072		.0177*	.9563	2.19
0.0005	9.668				.7693	.0105	.9618	2.38
0.001	9.175				.9390		.9563	14.79
0.002	10.17	.0642*			.5067		.9608	1.88
0.003	8.816	.0751		.2059*	.2904		.9643	1.64
0.0035	8.763	.0797		.2233*	.2459		.9633	1.77
0.0045	8.692	.0862		.2464*	.1865		.9618	1.94
0.01	8.560	.0991		.2883*	.0775		.9582	2.20
0.02	8.511	.1042		.3038*	.0368*		.9565	2.20
0.03+								

*See Table 5 for explanation.

+For value of $\lambda \geqslant 0.03$ the results are identical to $\lambda = 0.0$.

TABLE 11

REGRESSION COEFFICIENTS FOR THE MODEL $Q=Ae^{ut}K^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$
1940-1951

λ	$\log_e A$	α	β	γ	δ	u	R^2	lowest "t" value
0.0	9.551	.0742		.2768		.0099*	.9652	2.05
0.0005	9.668				.7693	.0105	.9618	2.38
0.0015	9.486				.8376		.9556	14.70
0.0025	9.411	.0660*		.1825*	.3004		.9691	1.81
0.0035	9.413	.0732		.2144*	.2087		.9686	2.24
0.004	9.413	.0756		.2245	.1801		.9683	2.32
0.0045	9.412	.0775		.2322	.1582*		.9681	2.30
0.01	9.406	.0858		.2647	.0662*		.9667	2.18
0.04	9.401	.0906		.2827	.0152*		.9658	2.10
0.09	9.404	.0912		.2854	.0064*		.9656	2.08

*See Table 5 for explanation.

TABLE 12

REGRESSION COEFFICIENTS FOR THE MODEL $Q=Ae^{ut}E^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}$
1940-1951

λ	$\log_e A$	α	β	γ	δ	u	R^2	lowest "t" value
0.0	9.809				.7190	.0157	.9630	4.36
0.0005	9.668				.7693	.0105	.9618	2.38
0.001	9.175				.9390		.9563	14.79
0.0025	8.140			.2302*	.5600		.9455	1.57
0.004	7.441			.3628	.3762		.9270	2.34
0.006	6.917			.4566	.2558		.9111	2.86
0.0065+	7.424			.4797		.0200	.9219	3.32

*See Table 5 for explanation.

+For values of $\lambda > 0.0065$, the results are identical.

TABLE 13

REGRESSION COEFFICIENTS FOR THE MODEL $Q=Ae^{ut}E^{\alpha}I^{\beta}P^{\gamma}W(\lambda)^{\delta}$
1940-1951

λ	$\log_e A$	α	β	γ	δ	u	R^2	lowest "t" value
0.002++	10.58	-.4306		.4938	.3041*	.0355	.9818	1.11
0.003	10.59	-.4629		.5500	.2405*	.0363	.9807	0.86
0.004	10.65	-.4839		.5991	.1502*	.0395	.9796	0.57
0.005	10.71	-.4905		.6261	.0763*	.0431	.9790	0.33
0.0055+	10.80	-.4835		.6369		.0479	.9786	-3.76

*See Table 5 for explanation.

++ For values of $\lambda < 0.002$ the results of this model are identical to those presented in Table 12 for equivalent values of λ.

+For values of $\lambda > 0.0055$ the results are the same as for $\lambda = 0.0055$.

TABLE 14

REGRESSION COEFFICIENTS FOR THE MODEL $Q=Ae^{ut}K^{\alpha}L^{\beta}N^{\gamma}W(\lambda)^{\delta}$
1952-1964

λ	$\log_e A$	α	β	γ	δ	u	R^2	lowest "t" value
0.0	4.762	.0636*		.7279			.9856	1.69
0.0005	7.047	.0875*		.3611	.3497*		.9888	1.43
0.001	7.027	.0853*		.3674*	.2618*		.9889	1.63
0.002	7.378	.0823*		.3887*	.1668*		.9889	1.62
0.003	7.392	.0807*		.4032*	.1206*		.9888	1.60
0.0045	7.381	.0793*		.4168*	.0845*		.9887	1.57
0.01	7.351	.0775*		.4354*	.0398*		.9886	1.53
0.03	7.312	.0763*		.4488*	.0133*		.9885	1.50
0.04+								

+For values of $\lambda \geqslant 0.04$ the results are identical to $\lambda = 0.0$.

*See Table 5 for explanation.

TABLE 15

REGRESSION COEFFICIENTS FOR THE MODEL $Q=Ae^{ut}K^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$
1952-1964

λ	$\log_e A$	α	β	γ	δ	u	R^2	lowest "t" value
0.0	7.359			.5945		.0170	.9848	2.45
0.001	8.439	.0755*		.2816*	.3426		.9892	1.61
0.002	8.661	.0699*		.3053*	.2226		.9894	1.77
0.003	8.754	.0671*		.3191*	.1638		.9894	1.71
0.0045	8.819	.0648*		.3312*	.1168		.9894	1.65
0.01	8.896	.0618*		.3478*	.0566		.9893	1.57
0.03	8.933	.0598*		.3594	.0193		.9892	1.52
0.07	7.928			.5375	.0091		.9865	2.82

*See Table 5 for explanation.

TABLE 16

REGRESSION COEFFICIENTS FOR THE MODEL $Q=Ae^{ut}E^{\alpha}L^{\beta}P^{\gamma}W(\lambda)^{\delta}$
1952-1964

λ	$\log_e A$	α	β	γ	δ	u	R^2	lowest "t" value
0.0	7.359			.8595		.0170	.9848	2.45
0.001	9.054		-.2174*	.5123	.2938		.9889	-1.79
0.0015	9.071		-.2021*	.5188	.2311		.9888	-1.65
0.0025	9.085		-.1853*	.5270	.1613		.9887	-1.50
0.003	7.566			.5281	.1497		.9860	2.71
0.006	7.747			.5308	.0845		.9863	2.78
0.009	7.806			.5323	.0618		.9864	2.80

*See Table 5 for explanation.

TABLE 17

EXTRAPOLATIONS OF THE THEIL-NAGAR AND DURBIN-WATSON TABLES FOR TESTING FOR AUTOCORRELATION OF REGRESSION DISTURBANCES[a]

	Number of Observations	Number of Variables 1	2	3	4
Theil-Nagar	12	1.32	1.53	1.88	2.17
	13	1.33	1.53	1.83	2.06
Durbin-Watson	12	1.34	1.57	1.86	2.15
	13	1.35	1.56	1.82	2.08

[a]Sources and method of extrapolation are given in Chapter V.

TABLE 18

REGRESSION COEFFICIENTS RECALCULATED FOR MODEL 3 AFTER ADJUSTMENTS FOR AUTOCORRELATION

λ	$\log_e A$	α	δ	γ	R^2	lowest "t" value
0.001	9.802		.7065		.9476	19.95
0.0015	8.194		.3767	.2708*	.9440	1.79
0.002	8.094		.2911	.3068	.9418	2.06

*See Table 5 for explanation.

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